Vectors

A *vector* is an ordered list of numbers , as **Z = [2,4,6,8] which called row vector and**

A = $\begin{bmatrix} 3 \\ 5 \\ 76 \end{bmatrix}$ **which called column vector.**

Matrices

A *matrix* is a set of numbers ordered in rows and columns vectors, as in the example. Consider the 3 ×3 matrix $A = \begin{bmatrix} 2 & 5 & 6 \\ -6 & 7 & 9 \\ 12 & 55 & 13 \end{bmatrix}$

Note that the matrix *elements* in any row are separated by commas, and can also be separated by spaces.

Definition1:-Thesize of a matrix is given by(number of its rows x number of its columns).

Definition2:-Two matrices A , B are equal if and only if they are from the same size and the symmetric elements are equal , in other words A = B $\Leftrightarrow a_{ij} = b_{ij} \; \forall$ i=1,2,..,m and j=1,2,3,…..,n .

The general form of a matrix from size mxn is written as:-

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & \cdots\cdots & a_{2n} \\ a_{31} & a_{32} & \cdots\cdots & \cdots\cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots\cdots & \cdots\cdots & a_{mn} \end{bmatrix} \text{ or } A = [a_{ij}] \quad , i=1,2,3,\ldots,m \; , j=1,2,3,\ldots,n \, .$$

Kinds of matrices :-

1)Zero matrix (Null):- all its elements equal zero , denoted by O .
2)Square matrix is a matrix which its rows equal to its columns .
3)Lower triangular matrix is a square matrix which all $a_{ij} = 0$, $\forall$ $i < j$.
4)Upper triangular matrix is a square matrix which all $a_{ij} = 0$, $\forall$ $i > j$.
5)Diagonal matrix is a square matrix which all a_{ij} not lies on main diagonal equal to zero, And may be denoted as diag(a_{11},a_{22},a_{33},……….,a_{nn}) .
6)Scalar matrix is a diagonal matrix which diag(k,k,k,…..,k) such that k is a constant number .
7)Identity matrix is a diagonal matrix which diag(1,1,1,…..,1) and denoted by I .

Examples:-

1) $A = \begin{bmatrix} 6 & 5 & 42 \\ 0 & 2 & 8 \\ 6 & -4 & 9 \\ 8 & 11 & -10 \end{bmatrix}$ is a matrix from size 4 x 3 .

2) $B = \begin{bmatrix} 2 & 43 & 5 & 4 \\ 0 & 0 & -5 & 6 \\ 3 & 22 & 3.4 & \sqrt{6} \\ 2.45 & 89 & \pi & 8 \end{bmatrix}$ is a square matrix from size 4 .

3) $G = \begin{bmatrix} 2 & 0 & 0 \\ -6 & 7 & 0 \\ 12 & 55 & 13 \end{bmatrix}$ is l0wer triangular matrix from size 3 , and

$F= \begin{bmatrix} 2 & 5 & 6 \\ 0 & 7 & 9 \\ 0 & 0 & 13 \end{bmatrix}$ is upper triangular matrix from size 3 .

4) $D= \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$ is a diagonal matrix from size 4 .

5) $S = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ is a scalar matrix from size 4 .

6) $I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ is Identity matrix from size 5 .

Operations on matrices :-

@1)*Addition of matrices* :- If two matrices **A** and **B** are from the same size, their (element-by-element) sum is obtained by typing **A + B** = C = [c_{ij}] , such that $c_{ij} = a_{ij} + b_{ij}, \forall\ i , j$.
Likewise, the difference of **A** and **B** represents by **A – B**= D =[a_{ij} - b_{ij}] , $\forall\ i , j$.

Example :- If $A = \begin{bmatrix} 6 & 5 \\ 7 & 0 \\ 8 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 \\ 8 & 3 \\ 14 & 7 \end{bmatrix}$, then $A + B = \begin{bmatrix} 8 & 10 \\ 15 & 3 \\ 22 & 8 \end{bmatrix}$.

$A - B = \begin{bmatrix} 4 & 0 \\ -1 & -3 \\ -6 & -6 \end{bmatrix}$, $B - A = \begin{bmatrix} -4 & 0 \\ 1 & 3 \\ 6 & 6 \end{bmatrix}$.

We can also add a scalar (a single number) to a matrix; **A + c** adds **c** to eachelement in **A**. Likewise, **A - c** subtracts the number **c** from eachelementof **A** , and c**A**multiply c by each element of **A** .

Example:- $A + 2 = \begin{bmatrix} 8 & 7 \\ 9 & 2 \\ 10 & 3 \end{bmatrix}$, $B - 4 = \begin{bmatrix} -2 & 1 \\ 4 & -1 \\ 10 & 3 \end{bmatrix}$, $5A = \begin{bmatrix} 30 & 25 \\ 35 & 0 \\ 40 & 5 \end{bmatrix}$.

Properties of matrices :-

For any matrices A , B , C , Zero matrix O from the same size ,and scalar numbers h , k :-
1) A + B = B + A .
2) A + (B + C) = (A + B) + C .
3) A + O = O + A = A .

4) A – A = O .
5) h(A + B) = hA + hB .
6) (h + k)A = hA + kA .
7) (hk)A = h(Ka) .
8) 1A = A , 0A = O .

@2) Multiplication of matrices :-

If **A** and **B** are multiplicatively compatible (that is, if **A** is $n\times m$ and **B** is $m\times$p), then their product **A*B** is $n\times p$. Recall that the element of **A*B** in the ith row and jth column is the sum of the products of the elements from the ith row of **A** times the elements from the jth column of **B**, that is,

$(\mathbf{A} * \mathbf{B})_{ij}=\mathbf{A}_{ik}\mathbf{B}_{kj},\quad 1 \le i \le n,\quad 1 \le j \le p.$

Example :-

If A = $\begin{bmatrix} 1 & 3 & -5 \\ 0 & 5 & 6 \\ 2 & -4 & 7 \end{bmatrix}$, B = $\begin{bmatrix} 5 & 2 & 0 & 3 \\ 2 & 11 & 8 & 1 \\ 6 & -2 & 4 & -1 \end{bmatrix}$, find AB , BA if possible .

Solution:-

a_{11} =[1 3 -5] $\begin{bmatrix} 5 \\ 2 \\ 6 \end{bmatrix}$ = 1*5+3*2+(-5)*6 = -19 , a_{12} =[1 3 -5] $\begin{bmatrix} 2 \\ 11 \\ -2 \end{bmatrix}$ = 1*2+3*11+(-5)*(-2) = 45,

a_{13} =[1 3 -5] $\begin{bmatrix} 0 \\ 8 \\ 4 \end{bmatrix}$ = 1*0+3*8+(-5)*4 = 4 , a_{14} =[1 3 -5] $\begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$ = 1*3+3*1+(-5)*(-1) = 11,

a_{21} =[0 5 6] $\begin{bmatrix} 5 \\ 2 \\ 6 \end{bmatrix}$ = 0*5+5*2+6*6 = 46 , a_{22} =[0 5 6] $\begin{bmatrix} 2 \\ 11 \\ -2 \end{bmatrix}$ = 0*2+5*11+6*(-2) = 43 ,

a_{23} = [0 5 6] $\begin{bmatrix} 0 \\ 8 \\ 4 \end{bmatrix}$ = 0*0+5*8+6*4 =6 4 , a_{24} =[0 5 6] $\begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$ = 0*3+5*1+6*(-1) = -1,

a_{31} =[2 -4 7] $\begin{bmatrix} 5 \\ 2 \\ 6 \end{bmatrix}$ = 2*5+(-4)*2+7*6 = 42 , a_{32} =[2 -4 7] $\begin{bmatrix} 2 \\ 11 \\ -2 \end{bmatrix}$ = 2*2+(-4)*11+7*(-2) = -54,

a_{33} =[2 -4 7] $\begin{bmatrix} 0 \\ 8 \\ 4 \end{bmatrix}$ = 2*0+(-4)*8+7*4 =- 4 , a_{34} =[2 -4 7] $\begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$ = 2*3+(-4)*1+7*(-1) = -5,

$\therefore$ AB = $\begin{bmatrix} -19 & 45 & 4 & 11 \\ 46 & 43 & 64 & -1 \\ 42 & -54 & -4 & -5 \end{bmatrix}$

BA is not possible because number of columns of B not equal to rows of A .

__Proposition__

1) If A is a square matrix from size n , then $AI_n = I_nA = A$.

2) If A(mxn) , B(nxp) , C(pxq) , then A(BC) = (AB)C .

Definition Let A is a matrix from size nxm , then the transpose of A is a matrix from size mxn denoted by A^T by changing rows with columns .

Example :-

If A = $\begin{bmatrix} 1 & 3 & -5 \\ 0 & 5 & 6 \\ 2 & -4 & 7 \end{bmatrix}$, B = $\begin{bmatrix} 5 & 2 & 0 & 3 \\ 2 & 11 & 8 & 1 \\ 6 & -2 & 4 & -1 \end{bmatrix}$, find A^T , B^T .

A^T = $\begin{bmatrix} 1 & 0 & 2 \\ 3 & 5 & -4 \\ -5 & 6 & 7 \end{bmatrix}$, B^T = $\begin{bmatrix} 5 & 2 & 6 \\ 2 & 11 & -2 \\ 0 & 8 & 4 \\ 3 & 1 & -1 \end{bmatrix}$

Proposition

1) If A , B two matrices from the same size then :- 11) $(A^T)^T = A$, 12) $(A+B)^T = A^T + B^T$
2) If A (mxn) , B(nxp) then $(AB)^T = B^T A^T$.

Example:-

Verify the proposition above for the matrices A = $\begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$, B = $\begin{bmatrix} 9 & 6 \\ 0 & 7 \end{bmatrix}$.

Definition:-

1)The square matrix A is called symmetric matrix if $A^T = A$, in other words $a_{ij} = a_{ji}$, $\forall$ $i \neq j$.
2)The square matrix A is called skew-symmetric matrix if $A^T = -A$, in other words $a_{ij} = -a_{ji}$, $\forall$ $i \neq j$ and the elements of main diagonal = 0 .

Examples:-

$A = \begin{bmatrix} 1 & 3 & -5 \\ 3 & 5 & 6 \\ -5 & 6 & 7 \end{bmatrix}$ is a symmetric matrix .

$F = \begin{bmatrix} 0 & -7 & 12 \\ 7 & 0 & -65 \\ -12 & 65 & 0 \end{bmatrix}$ is a skew-symmetric matrix .

Definition:- A square matrix A from size n is called (orthogonal matrix) if $AA^T = A^T A = I_n$.

Question :- Prove that A = $\begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$ is an orthogonal matrix ?

*Definition:-*Let A is a square matrix of size n , then the matrix B is called the invers matrix of A if and only if $AB = BA = I_n$ denoted by A^{-1}.

Notes

- Not for every square matrix an inverse .
- If A is an orthogonal matrix , then $A^T = A^{-1}$.

- IfA is a diagonal matrix such that D = $\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$, then

A^{-1} = $\begin{bmatrix} 1/a & 0 & 0 & 0 \\ 0 & 1/b & 0 & 0 \\ 0 & 0 & 1/c & 0 \\ 0 & 0 & 0 & 1/d \end{bmatrix}$.

Question :- If A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & \sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & 1/2 \end{bmatrix}$ orthogonal matrix , find A^{-1} ?

Proposition

If A ,B are two square matrices of size n and they have inverse for them ,then $(AB)^{-1} = B^{-1}A^{-1}$.

Proof :- $(AB).(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AA^{-1} = I_n$ …………………. (1)
$(B^{-1}A^{-1}).(AB) = B^{-1}(A^{-1}A)B = B^{-1}B = I_n$ …………………… (2)
$\therefore$ $(AB)^{-1} = B^{-1}A^{-1}$.

Question :- Prove that $(A^T)^{-1} = (A^{-1})^T$, if A is a square matrix and has an inverse ?

*Definition:-*For any square matrix A from size n there exist 0nly one number called the determinant of the matrix denoted by $|A|$.

Examples:-

$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ is a determinant from size 2 and its value = $a_{11} \cdot a_{22} - a_{21}a_{12}$.

$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ is a determinant from size 3 and its value calculate as in the below :-

$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$ = [main diagonals – secondary diagonals] =

$[a_{11}.a_{22}.a_{33} + a_{12}.a_{23}.a_{31} + a_{13}.a_{21}.a_{32}] - [a_{13}.a_{22}.a_{31} + a_{11}.a_{23}.a_{32} + a_{12}.a_{21}.a_{33}]$.

Definition :- The minor of $|A|$ is a determinant from $|A|$ after subtracting equal number from rows and columns of $|A|$.

The minor of a_{11} = $|M_{11}|$ = $\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$ and the minor of a_{32} = $|M_{32}|$ = $\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$

Definition :- The cofactor of the element a_{ij} = $(-1)^{i+j} . |M_{ij}|$ denoted by A_{IJ} .

Proposition

The value of any determinant equal to sum of multiplication elements of any rows(columns) by its cofactors .

Such that $|A| = a_{i1}A_{i1}+a_{i2}A_{i2}+a_{i3}A_{i3}+............+a_{in}A_{in}$, I = 1,2,3,……….,n OR

$= a_{1j}A_{1j}+a_{2j}A_{2j}+a_{3j}A_{3j}+............+a_{nj}A_{nj}$, j = 1,2,3,……….,n .

Example :- Find the value of $\begin{vmatrix} 1 & 4 & 9 & 2 \\ 2 & 0 & 3 & 0 \\ 5 & 0 & 0 & 7 \\ -3 & 0 & 9 & -2 \end{vmatrix}$

Proposition

If A is a square matrix and $|A| \neq 0$, then $A^{-1} = \frac{1}{|A|}.\left[A_{ij}\right]^T$.

Example:- Find A^{-1} of $\begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$, $\begin{vmatrix} 1 & 4 & 9 & 2 \\ 2 & 0 & 3 & 0 \\ 5 & 0 & 0 & 7 \\ -3 & 0 & 9 & -2 \end{vmatrix}$

Solving the system of linear equations by matrices

If we have the system of linear equations as below:-

$a_{11}x_1+a_{12}x_2+a_{13}x_3+.....................+a_{1n}x_n = b_1$

$a_{21}x_1+a_{22}x_2+a_{23}x_3+.....................+a_{2n}x_n = b_2$

$a_{31}x_1+a_{32}x_2+a_{33}x_3+.....................+a_{3n}x_n = b_3$

……………………………………………………..

……………………………………………………..

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$a_{m1}x_1+a_{m2}x_2+a_{m3}x_3+.....................+a_{mn}x_n = b_m$

Then $A = \begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & \cdots\cdots & a_{2n} \\ a_{31} & a_{32} & \cdots\cdots & \cdots\cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots\cdots & \cdots\cdots & a_{mn} \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ x_n \end{bmatrix}$, $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ . \\ . \\ b_m \end{bmatrix}$

$\therefore$ The solution to the system above is $X = A^{-1}.B$

Example :-

Solve the system of linear equations:-

2x+y=z

z-y+x=6

x+2y+z-3 =0

The solution

First :- we must arrange the equations as :-

2x+y-z=0

x-y+z=6
x+2y+z=3

A = $\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}$

$|A| = \left|\begin{array}{ccc|cc} 2 & 1 & -1 & 2 & 1 \\ 1 & -1 & 1 & 1 & -1 \\ 1 & 2 & 1 & 1 & 2 \end{array}\right.$ = -9 ≠ 0

A_{11} = $\begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix}$ = -3 , A_{12} = -1$\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$ = 0 , A_{13} = $\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}$ = 3

A_{21} = -1$\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$ = -3 , A_{22}= $\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$ = 3 , A_{23} = -1$\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$ = -3

A_{31} = $\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}$ = 0 , A_{32} = -1$\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$ = -3 , A_{33} = $\begin{vmatrix} 2 & 1 \\ 1 & - \end{vmatrix}$ = -3

$[A_{ij}]$ = $\begin{bmatrix} -3 & 0 & 3 \\ -3 & 3 & -3 \\ 0 & -3 & -3 \end{bmatrix}$, $[A_{ij}]^T$ = $\begin{bmatrix} -3 & -3 & 0 \\ 0 & 3 & -3 \\ 3 & -3 & -3 \end{bmatrix}$,

$\therefore$ A^{-1} = $\frac{1}{-9}\begin{bmatrix} -3 & -3 & 0 \\ 0 & 3 & -3 \\ 3 & -3 & -3 \end{bmatrix}$

$\therefore$ X = $\frac{1}{3}\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}$ = $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

Questions

1)If A = $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 5 \\ 6 & 7 & -2 \end{bmatrix}$,B = $\begin{bmatrix} 9 & 0 & 12 \\ 5 & 6 & 1 \\ -1 & 0 & 0 \end{bmatrix}$, Find (1) 2A – 3B , (2) I_3 + 4B - 3B

(3) AB , BA , what do you notice ?

2) Write the matrix A from size 3x4 such that A = $\begin{cases} 7 & , \quad \forall i < j \\ 3 & , \quad \forall i = j \\ j - i & , \quad \forall i > j \end{cases}$

3) Find the value of x , y , z , t if $\begin{bmatrix} 3x & 1 \\ z & 3+2t \end{bmatrix}$ - 2$\begin{bmatrix} x & -y \\ 3z & -2t \end{bmatrix}$ = 3$\begin{bmatrix} -1 & x-y \\ x+y & 2z \end{bmatrix}$

4) If A = $\begin{bmatrix} 2 & 4 & 1 \\ -1 & 3 & -2 \\ 2 & -3 & 5 \end{bmatrix}$, B = $\begin{bmatrix} -11 \\ -16 \\ 21 \end{bmatrix}$, find the matrix X which satisfy AX = B .

5) If A $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & \sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & 1/2 \end{bmatrix}$ is orthogonal matrix , find A^{-1} .

6) Write a symmetric matrix from size 4 .

7) Write a skew-symmetric matrix from size 5 .

8) Is there exist an inverse matrix for A = $\begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$?

9) S0lve the following system of linear equations:-

(1)
$2x_1 - 4x_2 - x_3 = 2$
$3x_2 - 2x_3 + x_1 = 0$
$-6 + 3x_1 = 2x_2 + 3x_3$

(2)
$10x_3 + 6x_2 = 9 - 3x_1$
$x_1 + x_2 = 4 - x_3$
$3x_2 + 2x_1 + 4x_3 = 0$

10) Find the value of $\begin{vmatrix} 4 & 5 & -6 & -1 \\ 2 & 8 & 5 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & -2 \end{vmatrix}$

11) If A = $\begin{bmatrix} 2 & -2 & 1 \\ 0 & 4 & 0 \\ 11 & 8 & -5 \end{bmatrix}$, B = $\begin{bmatrix} 8 & 7 \\ 9 & 2 \\ 10 & 3 \end{bmatrix}$, C = $\begin{bmatrix} 0 & 90 \\ 2 & 4 \\ -4 & 12 \end{bmatrix}$, Is A .(B + C) = A.B + A.C ?

12) Full the following statements with suitable words :-

a) Tow matrices are equal if and only if ………………….. .

b) Lower triangular matrix is ……………………………….. .

c) Upper triangular matrix is ……………………………….. .

d) Diagonal matrix is ………………………………………… .

e) We can multiply the matrix A by the matrix B if ……. .

f) A is a symmetric matrix if and only if ……………………. .

g) IF the matrix $A = -A^T$ then A is called………………….. .

h) If $AB = BA = I_n$ then B is called ………………………… .

t) The minor to $a_{ij}$of the determinant A is …………………..

k) The cofactor of the element a_{ij}denoted by …….. and equal to ………… .

13) For the matrices A = $\begin{bmatrix} 2 & -2 & 1 \\ 0 & 4 & 0 \\ 11 & 8 & -5 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$, C = $\begin{bmatrix} 1 & 0 & 2 \\ 3 & 5 & -4 \\ -5 & 6 & 7 \end{bmatrix}$

Verify the following :-

1- A + B = B + A
2- A + (B + C) = (A + B) + C
3- A + O = O + A = A
4- C – C = O
5- $(B^T)^T = B$
6- $AI_3 = I_3A = A$
7- A(BC) = (AB)C
8- $(B + C)^T = B^T + C^T$

Functions and differentiation