L7

Node and supernode analysis

procedure !

(1) Select a node as the refrence node

(2) Assign voltages $v_1, v_2, \ldots$ to the remaining nodes , The voltages are refrenced with respect to the refrence node

(3) Apply KCL to each node exept the refrence node

(4) Use Ohm's Law to express the branch currents in terms of the node voltages.

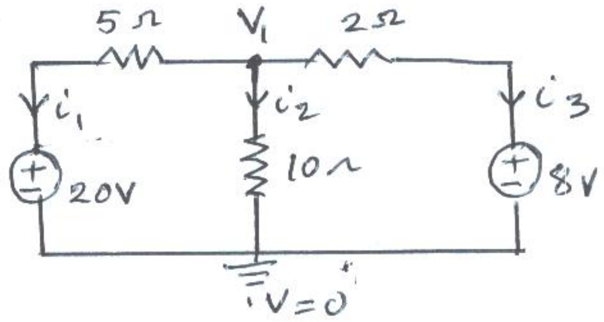
(5) Solve the resulting simultaneous equations to obtain the unknown node voltages

we usually take the ground node as the refrence node with $V=0$

$V=0$

Examples

Example 1 : single node



KCL at node (1)

$$i_1 + i_2 + i_3 = 0 \quad \text{----} (1)$$

$$\left.\begin{aligned} i_1 &= \frac{V_1 - 20}{5} \\ i_2 &= \frac{V_1}{10} \\ i_3 &= \frac{V_1 - 8}{2} \end{aligned}\right\} \quad \text{----} (2)$$

using (2) in (1):

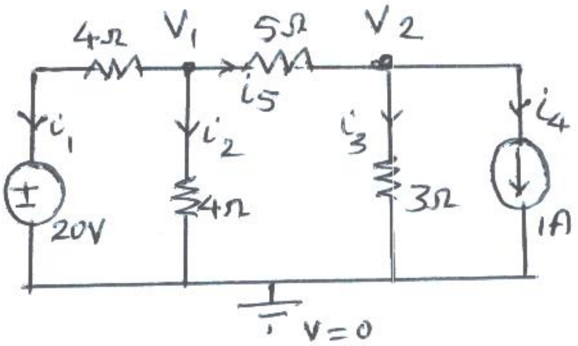
$$\frac{V_1 - 20}{5} + \frac{V_1}{10} + \frac{V_1 - 8}{2} = 0$$

$$V_1\left(\frac{1}{5} + \frac{1}{10} + \frac{1}{2}\right) = \frac{20}{5} + \frac{8}{2} = 4 + 4 = 8$$

$$V_1\left(\frac{2+1+5}{10}\right) = 8$$

$$\boxed{V_1 = 10\ V}$$

Example 2 : 2 nodes



Using nodal analysis, find V_1 and V_2 at nodes indicated (1) and (2) in the circuit shown in the Figure

solution :

KCL at node 1

$$i_1 + i_2 + i_5 = 0 \quad \text{--- (1)}$$

$$i_1 = \frac{V_1 - 20}{4}, \quad i_2 = \frac{V_1}{4}, \quad i_5 = \frac{V_1 - V_2}{5} \quad \text{--- (2)}$$

sub. (2) in (1) :

$$\frac{V_1 - 20}{4} + \frac{V_1}{4} + \frac{V_1 - V_2}{5} = 0$$

$$V_1\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{5}\right) - \frac{V_2}{5} = 5 \Rightarrow V_1\left(\frac{1}{2} + \frac{1}{5}\right) - \frac{V_2}{5} = 5$$

$$\frac{7}{10}V_1 - \frac{V_2}{5} = 5$$

$$7V_1 - 2V_2 = 50 \quad \text{--- (3)}$$

KCL at node 2

$$i_3 + i_4 - i_5 = 0 \quad \text{--- (4)}$$

$$i_3 = \frac{V_2}{3} \quad , \quad i_4 = 1 \quad \text{-----} (5)$$

Using (5) in (4)

$$\frac{V_2}{3} + 1 - \frac{V_1 - V_2}{5} = 0$$

$$-3V_1 + 8V_2 = -15 \quad \text{-----}(6)$$

$$D = \begin{vmatrix} 7 & -2 \\ -3 & 8 \end{vmatrix} = 56 - 6 = 50$$

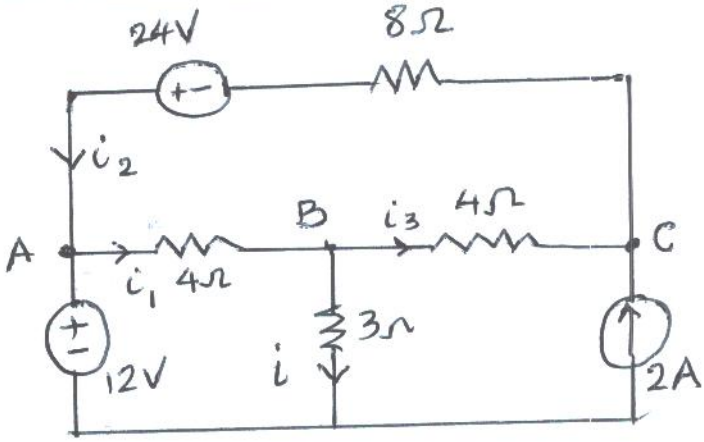
$$V_1 = \frac{1}{50} \begin{vmatrix} 50 & -2 \\ -15 & 8 \end{vmatrix} = \frac{1}{50}(400 - 30) = \frac{37}{5}\ V$$

$$V_2 = \frac{1}{50} \begin{vmatrix} 7 & 50 \\ -3 & -15 \end{vmatrix} = \frac{1}{50}(-105 + 150)$$

$$= \frac{45}{50} = 0.9\,V$$

Example 3 : 3 nodes

Find nodal voltages V_A, V_B and V_C in the circuit shown in the Figure.



Solution :

$V_A = 12\ V$

$$i_1 = \frac{V_A - V_B}{4} = \frac{12 - V_B}{4} \quad \text{----} (1)$$

$$i_2 = \frac{V_C + 24 - V_A}{8} = \frac{V_C - 36}{8} \quad \text{---} (2)$$

$$i = \frac{V_B}{3} \quad , \quad i_3 = \frac{V_B - V_C}{4} \quad \text{---} (3)$$

KCL at B

$i_1 = i_3 + i$

$$\frac{12 - V_B}{4} = \frac{V_B - V_C}{4} + \frac{V_B}{3}$$

$$\frac{5}{6} V_B - \frac{1}{4} V_C = 3 \qquad \text{-----}(4)$$

KCL at C :

$2 = i_3 + i_2$

$$2 = \frac{V_B - V_C}{4} + \frac{V_C - 36}{8}$$

simplify :

$$2 V_B - V_C = -20 \quad ** \qquad \text{----}(5)$$

$$V_A + 0 V_B + 0 V_C = 12 \quad * \qquad \text{---}(6)$$

$$10 V_B - 3 V_C = 36 \quad *** \qquad \text{----}(4)$$

$$D = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 10 & -3 \end{vmatrix} = -6 + 10 = 4$$

$$V_A = \frac{1}{4} \begin{vmatrix} +12 & 0 & 0 \\ -20 & 2 & -1 \\ 36 & 10 & -3 \end{vmatrix} = \frac{1}{4} (+120)(-6 + 10) = +120$$

$$V_B = \frac{1}{4} \begin{vmatrix} 1 & +12 & 0 \\ 0 & -20 & -1 \\ 0 & 36 & -3 \end{vmatrix} = \frac{1}{4} (+60 + 36) - 12 \times 0$$

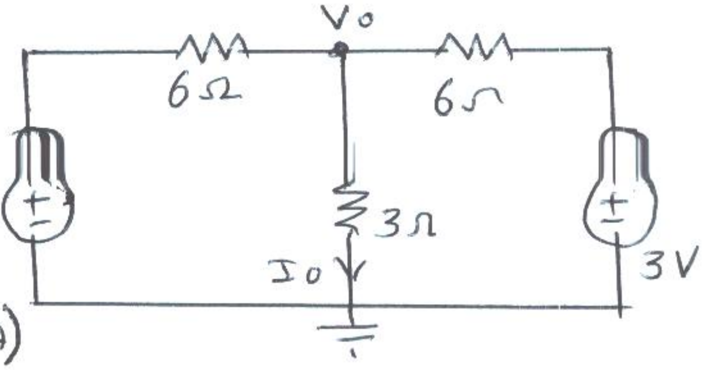
$$= \frac{1}{4} (96) = 24 \text{ V}$$

$$V_C = \frac{1}{4} \begin{vmatrix} 1 & 0 & 12 \\ 0 & 2 & -20 \\ 0 & 10 & 36 \end{vmatrix} = \frac{(72 + 200)}{4} = \frac{272}{4} = 68 \text{ V}$$

Home work

Prob. 1 :

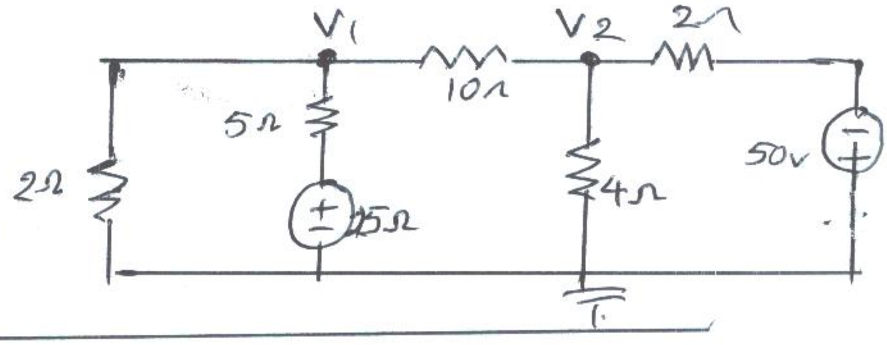
Find I_o in the circuit shown in the Fig using nodal analysis. (Ans: $I_o = \frac{3}{4}A$)



Prob. 2 :

Find V_1 and V_2 using nodal analysis

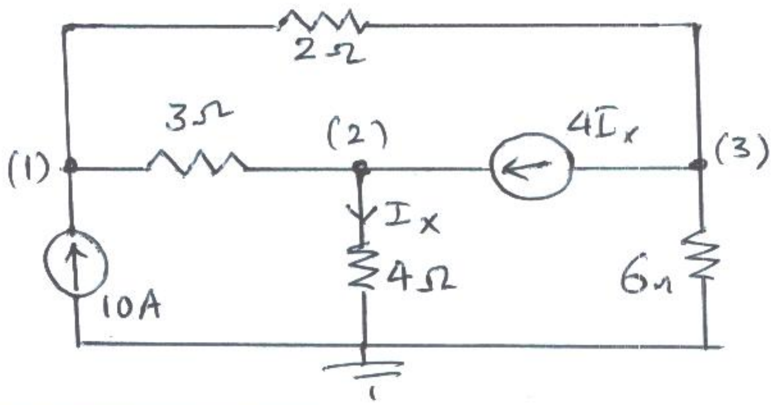
(Ans: $V_1 \simeq 2.6V$, $V_2 \simeq 29V$)



Prob. 3 :

Find V_1, V_2 and V_3 in the Fig.

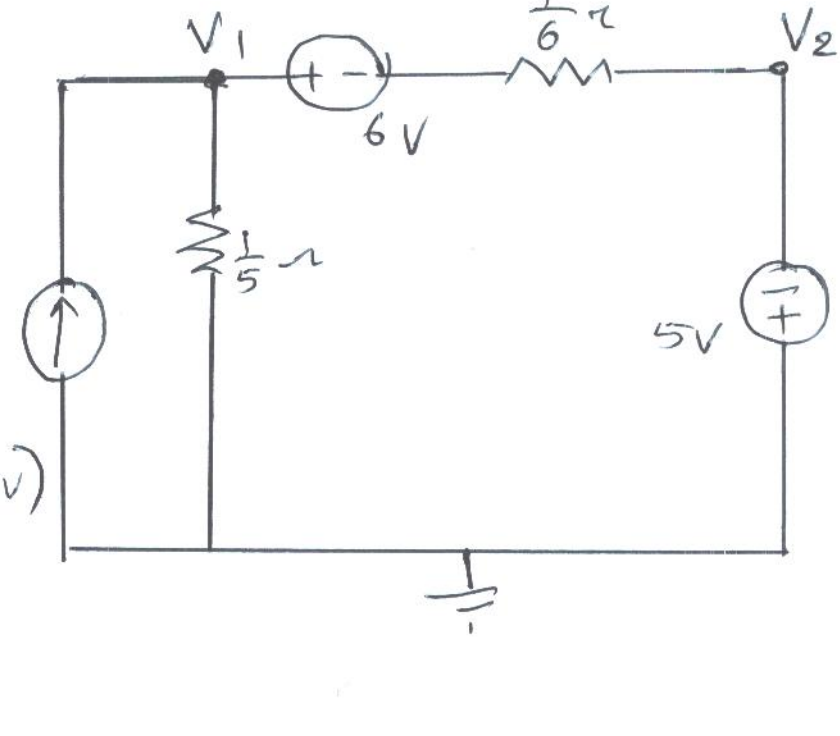
(Ans: $V_1 = 80V$, $V_2 \simeq -64V$, $V_3 \simeq 156V$)



Prob 4 :

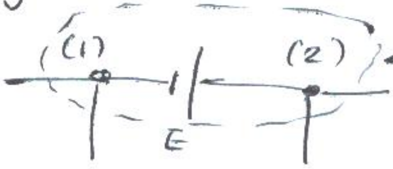
Find V_1 and V_2 using nodal analysis

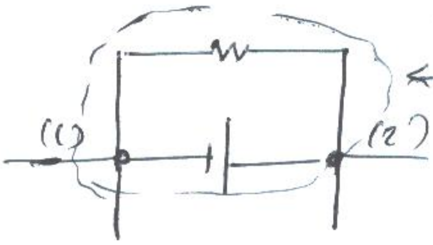
(Ans: $V_1 = 3.38V$, $V_2 = -5V$)



Supernode analysis

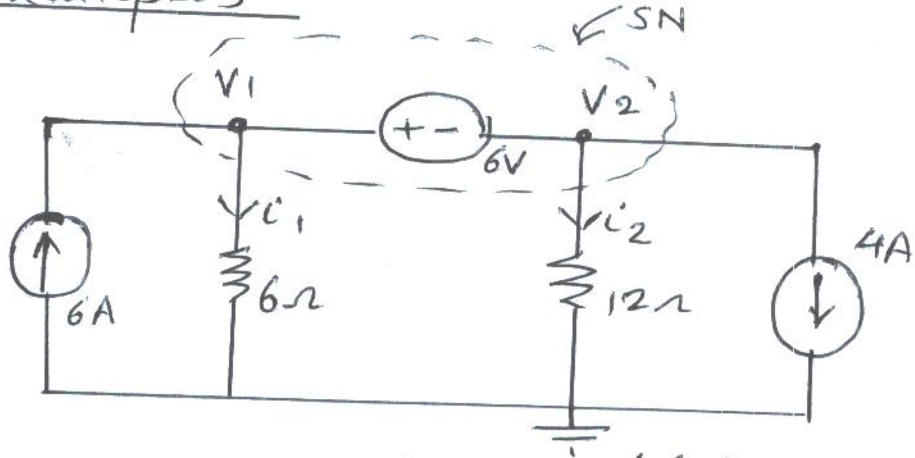
A supernode is formed by enclosing a voltage source connected between two nonrefrence nodes and any elements connected in parallel with it





Examples

Example 1 :
Determine V_1 and V_2 in the Circuit shown in the Figure



Solution :

KCL for SN : $\sum_{SN} i = 0$

$i_1 + i_2 - 6 + 4 = 0$

$i_1 + i_2 = 2$ ----- (1)

$i_1 = \frac{V_1}{6}$, $i_2 = \frac{V_2}{12}$ ----- (2)

using (2) in (1)

$$\frac{V_1}{6} + \frac{V_2}{12} = 2$$

$2V_1 + V_2 = 24$ ----- (3)

Inside SN : $V_1 - V_2 = 6$ --- (4)

solve (3) and (4)

$$D = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -3$$

$$V_1 = \begin{vmatrix} 24 & 1 \\ 6 & -1 \end{vmatrix} \frac{1}{D}$$

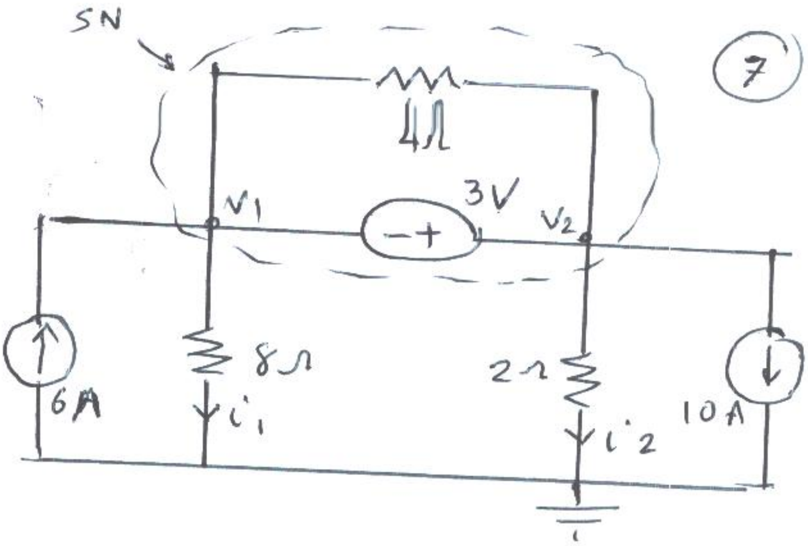
$$= -\frac{1}{3}[-24-6] = 10V$$

$$V_2 = -\frac{1}{3}\begin{vmatrix} 2 & 24 \\ 1 & 6 \end{vmatrix}$$

$$= -\frac{1}{3}(12-24) = 4V$$

Example 2 :

Determine V_1 and V_2 in the circuit shown in the Fig using the concept of a supernode



Solution

apply KCL on SN $\quad \sum_{SN} i = 0$

$$6 = i_1 + i_2 + 10$$

$$\therefore \; i_1 + i_2 = -4 \qquad \text{----(1)}$$

$$i_1 = \frac{V_1}{8} \;,\; i_2 = \frac{V_2}{2} \qquad \text{---(2)}$$

Using (2) in (1) :

$$\frac{V_1}{8} + \frac{V_2}{2} = -4 \qquad \text{----(3)}$$

$$\therefore \; V_1 + 4V_2 = -32 \qquad \text{----(3)}$$

Inside SN :

$$V_2 - V_1 = 3 \qquad \text{---(4)}$$

$$D = \begin{vmatrix} 1 & 4 \\ -1 & 1 \end{vmatrix} = +5$$

$$V_1 = \begin{vmatrix} -32 & 4 \\ 3 & 1 \end{vmatrix} \left(\frac{1}{+5}\right) = +\frac{1}{5}\left(-32-12\right) = -\frac{44}{5} \text{ V}$$

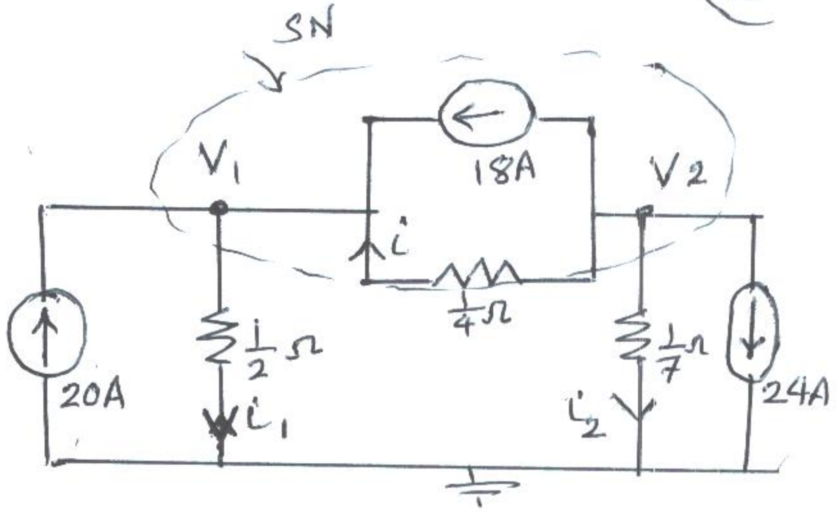
$$V_1 = -8.8\text{V}$$

$$V_2 = \begin{vmatrix} 1 & -32 \\ -1 & 3 \end{vmatrix} \left(\frac{1}{5}\right) = \frac{1}{5}\left(3-32\right) = -\frac{29}{5} = -5.8\text{V}$$

$$\therefore i_1 = \frac{V_1}{\ldots} = -1.1\text{A} \;,\; i_2 = \frac{V_2}{\ldots} = -2.9\text{A}$$

Example 3

Determine V_1 and V_2 in the circuit shown in the Figure



Solution :

Current entering SN = Current leaving SN

$$24 + 7V_2 + 2V_1 = 20$$

$$\therefore \; 2V_1 + 7V_2 = -4 \quad \text{---} \; (1)$$

KCL at node 1 :

$$20 - i_1 + 18 + i = 0$$

$$38 \cancel{+i} - 2V_1 \cancel{+} 4(V_1 - V_2) = 0$$

$$38 = 6V_1 - 4V_2$$

$$3V_1 - 2V_2 = 19 \quad \text{---} \; (2)$$

$$D = \begin{vmatrix} 2 & 7 \\ 3 & -2 \end{vmatrix} = -4 - 21 = -25$$

$$V_1 = \begin{vmatrix} -4 & 7 \\ 19 & -2 \end{vmatrix} \frac{1}{D} = (8 - 133)\left(\frac{1}{-25}\right)$$

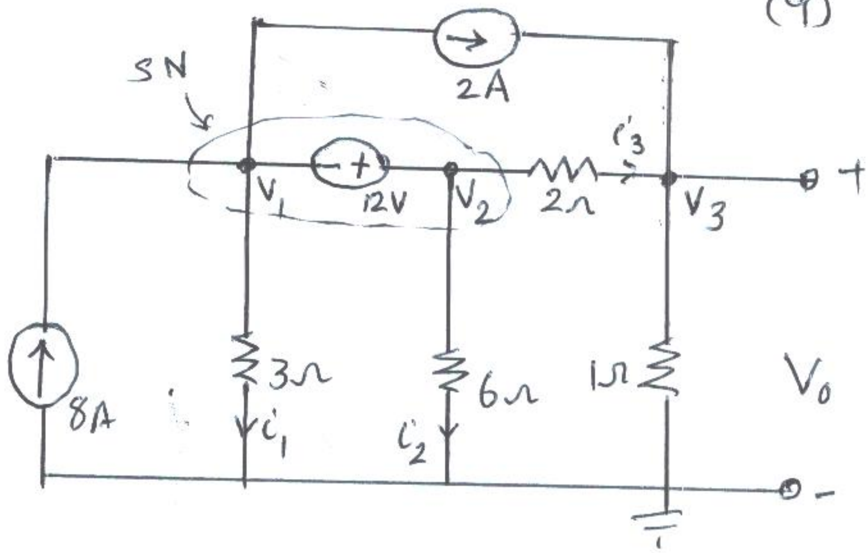
$$= -\frac{1}{25}(-125) = 5\,V$$

$$V_2 = \frac{-1}{25} \begin{vmatrix} 2 & -4 \\ 3 & 19 \end{vmatrix} = -\frac{1}{25}(38 + 12) = \frac{-50}{25} = -2\,V$$

Example 4

Find V_o using nodal analysis

Solution:



Apply KCL on SN

$$-8 + i_1 + i_2 + i_3 + 2 = 0$$

$$-8 + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2 - V_3}{2} + 2 = 0$$

Simplify

$$2V_1 + V_2 + 3V_2 - 3V_3 = 36$$

$$2V_1 + 4V_2 - 3V_3 = 36 \quad \text{--- (1)}$$

Inside SN

$$V_2 - V_1 = 12 \quad \text{--- (2)}$$

KCL on node 3

$$\frac{V_2 - V_3}{2} + 2 - \frac{V_3}{1} = 0$$

$$V_2 - 3V_3 + 4 = 0$$

$$V_2 - 3V_3 = -4 \quad \text{--- (3)}$$

collect eqs

$$2V_1 + 4V_2 - 3V_3 = 36$$
$$-V_1 + V_2 + 0V_3 = 12$$
$$0V_1 + V_2 - 3V_3 = -4$$

$$\Rightarrow D = \begin{vmatrix} 2 & 4 & -3 \\ -1 & 1 & 0 \\ 0 & 1 & -3 \end{vmatrix} = 2(-3) - 4(3) - 3(-1)$$
$$= -6 - 12 + 3$$
$$= -15$$

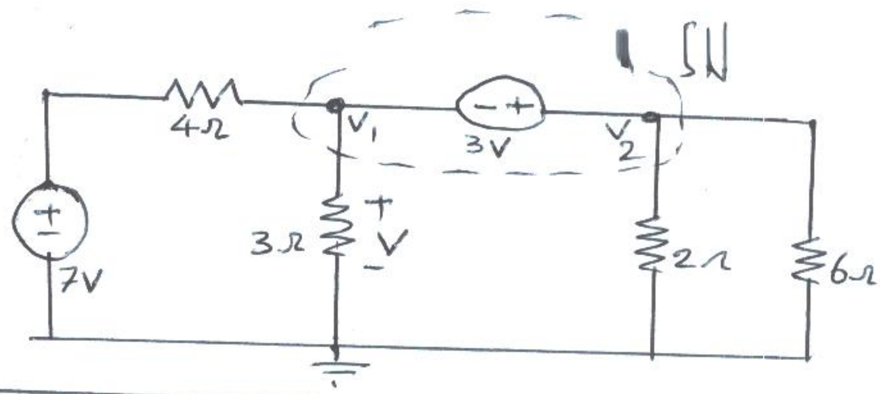
$$V_3 = \left(\frac{1}{-15}\right) \begin{vmatrix} 2 & 4 & 36 \\ -1 & 1 & 12 \end{vmatrix} = \ldots$$

Home work

Problem (1)

Determine V_1, V_2

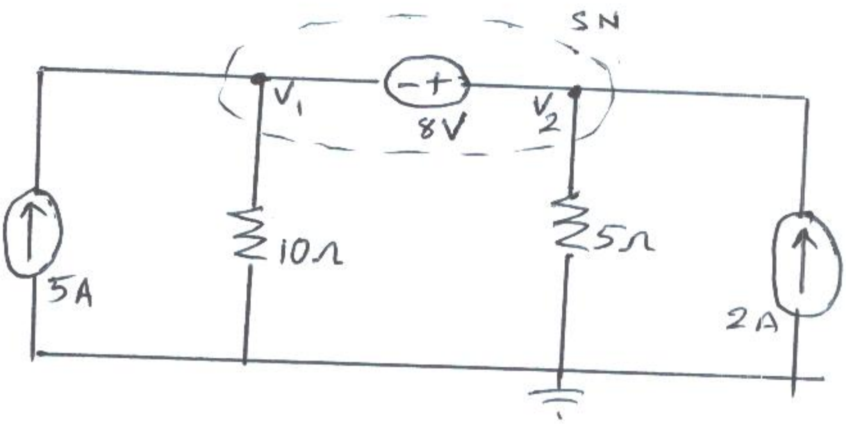
[Ans : $V_1 = -0.2V$
$V_2 = (14/5)V$]



Problem (2) :

Determine V_1, V_2

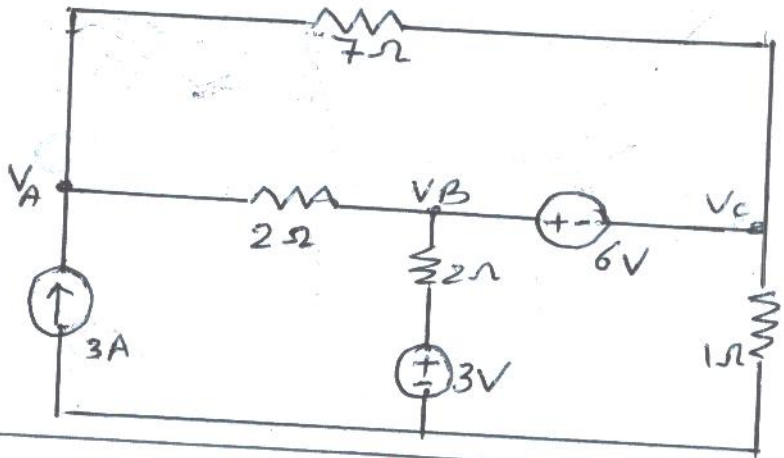
[Ans : { $V_1 = 18V$
$V_2 = 26V$]



Problem (3) :

Find V_A, V_B and V_C

Ans [$V_A = 10.33V$
$V_B = 7V$
$V_C = 1V$]



Problem(4)

Ans [$V_1 = -7.333V$
$V_2 = -5.333V$]