3.3 Current Sources

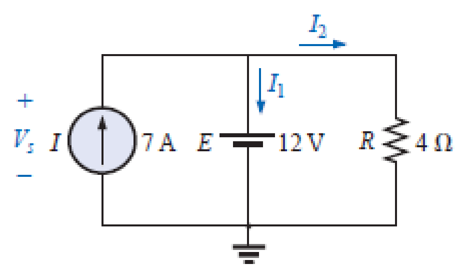
The current source is often referred to as the *dual* of the voltage source. A battery supplies a *fixed* voltage, and the source current can vary; but the current source supplies a *fixed* current to the branch in which it is located, while its terminal voltage may vary as determined by the network to which it is applied. Note from the above that *duality* simply implies an interchange of current and voltage to distinguish the characteristics of one source from the other.

A current source determines the current in the branch in which it is located

and

the magnitude and polarity of the voltage across a current source are a function of the network to which it is applied.

Example 3.7 Find the voltage *Vs* and the currents I_1 and I_2 ??



Solution:

$$V_s = E = \mathbf{12\ V}$$

$$I_2 = \frac{V_R}{R} = \frac{E}{R} = \frac{12\ \text{V}}{4\ \Omega} = \mathbf{3\ A}$$

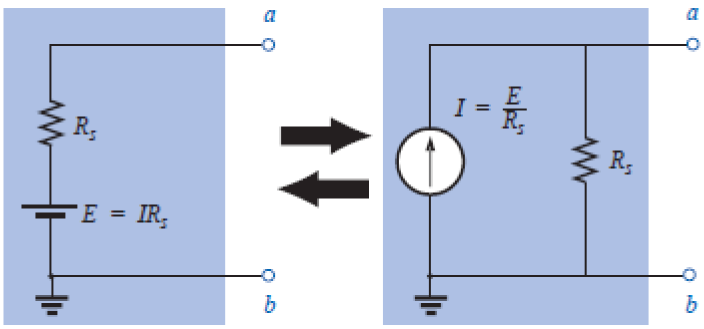
Applying Kirchhoff's current law:

$$I = I_1 + I_2$$

and

$$I_1 = I - I_2 = 7\ \text{A} - 3\ \text{A} = \mathbf{4\ A}$$

3.4 Source Conversions



Example 3.8

a. Convert the voltage source of Fig.(a) below to a current source, and calculate the current through the 4-Ω load for each source.
b. Replace the 4-Ω load with a 1-kΩ load, and calculate the current I_L for the voltage source.

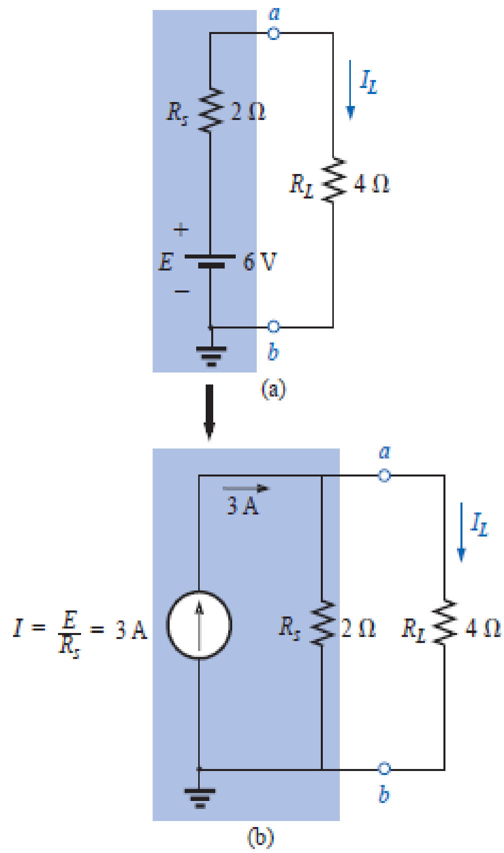
Solutions:

a.

$$I_L = \frac{E}{R_s + R_L} = \frac{6\text{ V}}{2\,\Omega + 4\,\Omega} = \mathbf{1\ A}$$

$$I_L = \frac{R_s I}{R_s + R_L} = \frac{(2\,\Omega)(3\text{ A})}{2\,\Omega + 4\,\Omega} = \mathbf{1\ A}$$

b. $I_L = \dfrac{E}{R_s + R_L} = \dfrac{6\text{ V}}{2\,\Omega + 1\text{ k}\Omega} \cong \mathbf{5.99\ mA}$



Example 3.9 Reduce the network of Fig. below to a single current source, and calculate the current through R_L ??

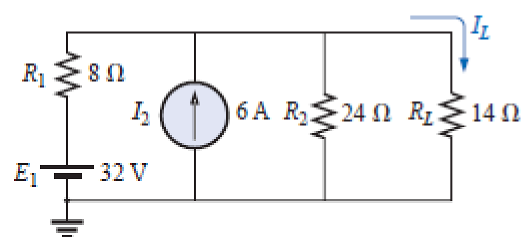
Solution

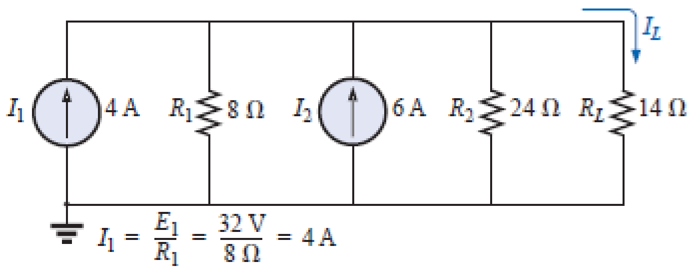
$$I_s = I_1 + I_2 = 4\text{ A} + 6\text{ A} = \mathbf{10\ A}$$

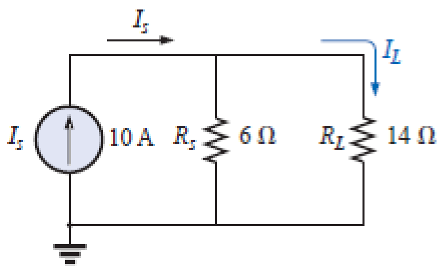
and $$R_s = R_1 \parallel R_2 = 8\,\Omega \parallel 24\,\Omega = \mathbf{6\,\Omega}$$

Applying the current divider rule

$$I_L = \frac{R_s I_s}{R_s + R_L} = \frac{(6\,\Omega)(10\text{ A})}{6\,\Omega + 14\,\Omega} = \frac{60\text{ A}}{20} = \mathbf{3\ A}$$







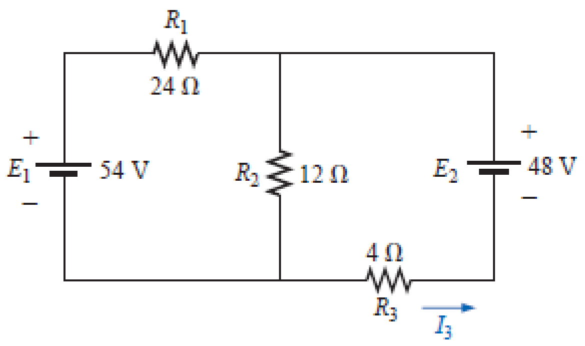
3.5 Superposition Theorem

The **superposition theorem,** like the methods of the last chapter, can be used to find the solution to networks with two or more sources that are not in series or parallel. The most obvious advantage of this method is that it does not require the use of a mathematical technique such as determinants to find the required voltages or currents. Instead, each source is treated independently, and the algebraic sum is found to determine a particular unknown quantity of the network.

The superposition theorem states the following:

The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.

Example 3.10 Using superposition, determine the current through the 4-Ω resistor ??



Solution: *Considering the effects of a 54-V source*

$$R_T = R_1 + R_2 \parallel R_3 = 24\ \Omega + 12\ \Omega \parallel 4\ \Omega = 24\ \Omega + 3\ \Omega = 27\ \Omega$$

$$I = \frac{E_1}{R_T} = \frac{54\ \text{V}}{27\ \Omega} = 2\ \text{A}$$

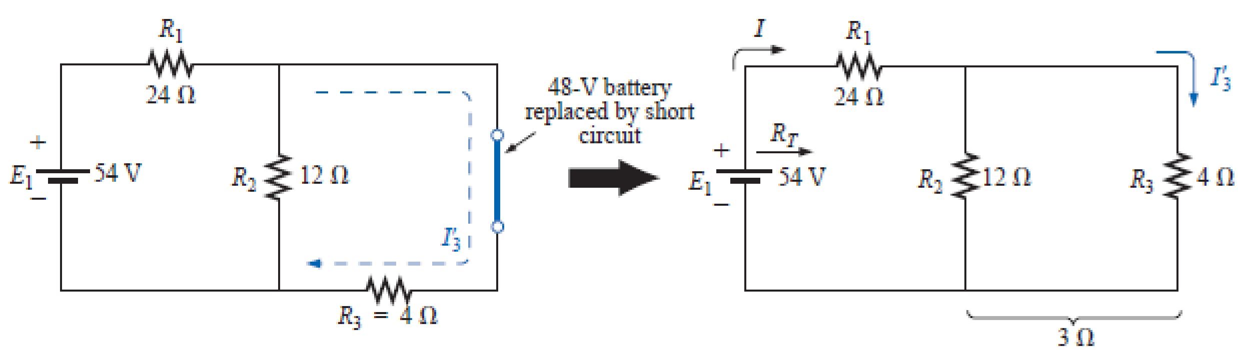
Using the current divider rule,

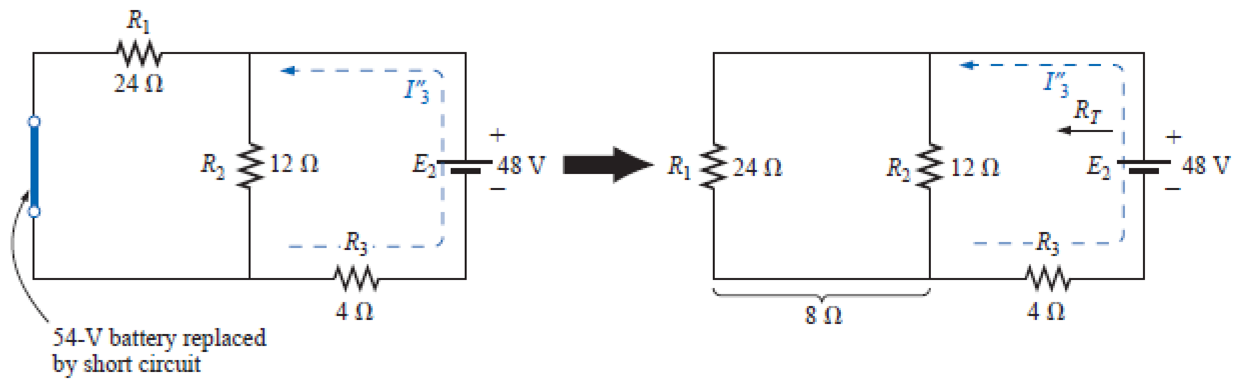
$$I'_3 = \frac{R_2 I}{R_2 + R_3} = \frac{(12\ \Omega)(2\ \text{A})}{12\ \Omega + 4\ \Omega} = \frac{24\ \text{A}}{16} = 1.5\ \text{A}$$

Considering the effects of the 48-V source

$$R_T = R_3 + R_1 \parallel R_2 = 4\ \Omega + 24\ \Omega \parallel 12\ \Omega = 4\ \Omega + 8\ \Omega = 12\ \Omega$$

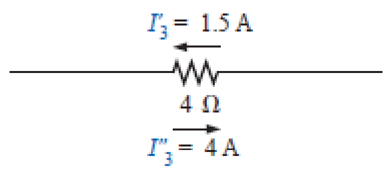
$$I''_3 = \frac{E_2}{R_T} = \frac{48\ \text{V}}{12\ \Omega} = 4\ \text{A}$$





The total current through the 4-Ω resistor

$$I_3 = I''_3 - I'_3 = 4\text{ A} - 1.5\text{ A} = \mathbf{2.5\text{ A}} \qquad (\text{direction of } I''_3)$$



3.6 Maximum Power Transfer Theorem

The **maximum power transfer theorem** states the following:

A load will receive maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thévenin resistance of the network as "seen" by the load.

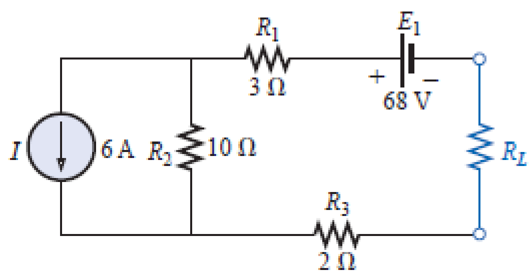
For the **Thévenin** circuit

$$\boxed{P_{L_{\max}} = \frac{E_{Th}^2}{4R_{Th}}} \qquad (\text{watts, W})$$

For the Norton circuit

$$\boxed{P_{L_{\max}} = \frac{I_N^2 R_N}{4}} \qquad (\text{W})$$

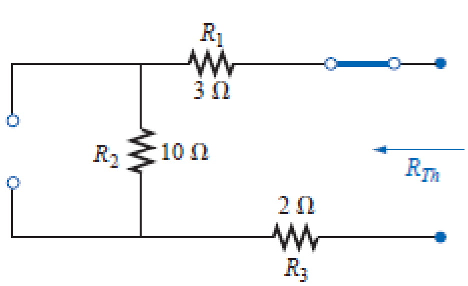
Example 3.11 Find the value of R_L in Fig. below for maximum power to R_L, and determine the maximum power ??



Solution

$$R_{Th} = R_1 + R_2 + R_3 = 3\ \Omega + 10\ \Omega + 2\ \Omega = 15\ \Omega$$

and $$R_L = R_{Th} = \mathbf{15\ \Omega}$$



$$V_1 = V_3 = 0\ \text{V}$$

and $$V_2 = I_2R_2 = IR_2 = (6\ \text{A})(10\ \Omega) = 60\ \text{V}$$

Applying Kirchhoff's voltage law,

$$\Sigma_{\circlearrowright} V = -V_2 - E_1 + E_{Th} = 0$$

and $$E_{Th} = V_2 + E_1 = 60\ \text{V} + 68\ \text{V} = 128\ \text{V}$$

Thus, $$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(128\ \text{V})^2}{4(15\ \Omega)} = \mathbf{273.07\ W}$$