3.1 The′venin's Theorem

The′venin's Theorem states the following: ***Any two-terminal, linear bilateral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor, as shown below:***

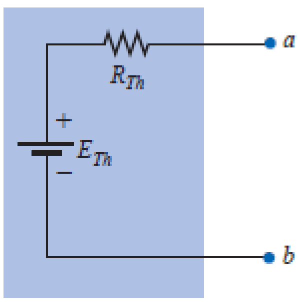


Fig.1 Thévenin equivalent circuit.

The following sequence of steps will lead to the proper value of R_{Th} and E_{Th}. Preliminary:

R_{Th}

1. Remove that portion of the network across which the Thévenin equivalent circuit is to be found. In Fig.1, this requires that the load resistor RL be temporarily removed from the network.

2. Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.) .
3. Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

E_{Th}:

4. Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that will lead to the most confusion and errors. In all cases, keep in mind that it is the open-circuit potential between the two terminals marked in step 2.) Conclusion:

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor RL between the terminals of the Thévenin equivalent circuit as shown in Fig. below.

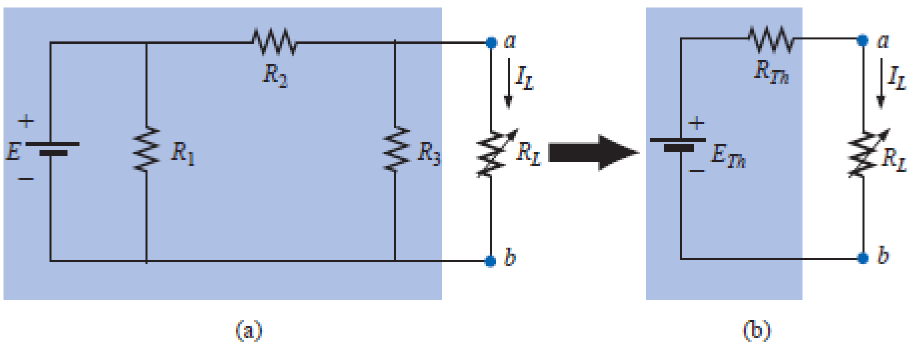
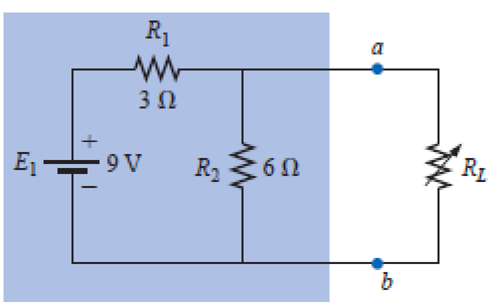


Fig.2 Substituting the Thévenin equivalent circuit for a complex network

Example 3.1 Find the Thévenin equivalent circuit for the network in the shaded area of the network of Fig. below. Then find the current through R_L for values of (2 ,10 ,and 100 Ω) **?**



Solution:

$$R_{Th} = R_1 \,||\, R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \mathbf{2\ \Omega}$$

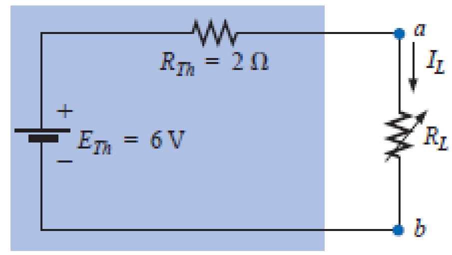
$$E_{Th} = \frac{R_2 E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = \mathbf{6\ V}$$

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

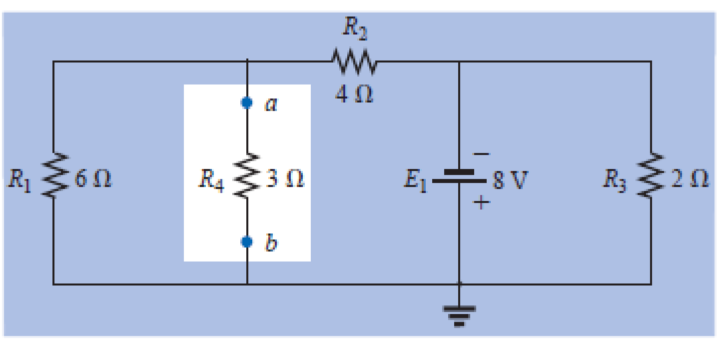
$R_L = 2\ \Omega$: $$I_L = \frac{6\ \text{V}}{2\ \Omega + 2\ \Omega} = \mathbf{1.5\ A}$$

$R_L = 10\ \Omega$: $$I_L = \frac{6\ \text{V}}{2\ \Omega + 10\ \Omega} = \mathbf{0.5\ A}$$

$R_L = 100\ \Omega$: $$I_L = \frac{6\ \text{V}}{2\ \Omega + 100\ \Omega} = \mathbf{0.059\ A}$$

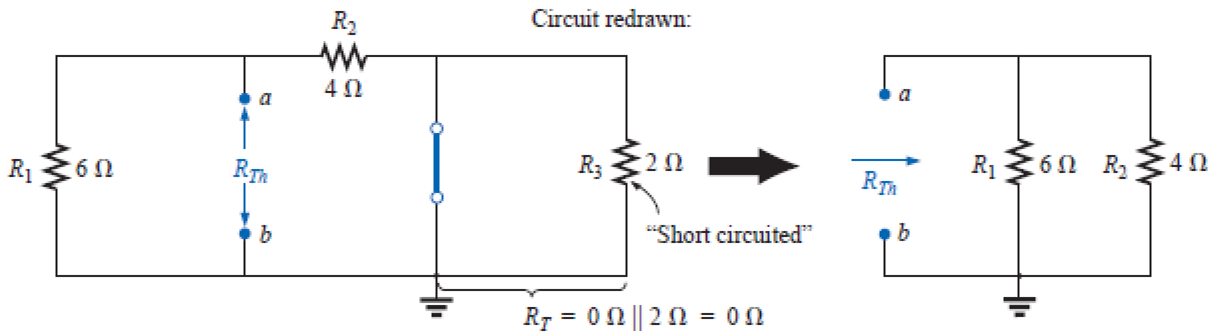


Example 3.2 Find the Thévenin equivalent circuit for the network in the shaded area of the network of Fig. below. ?

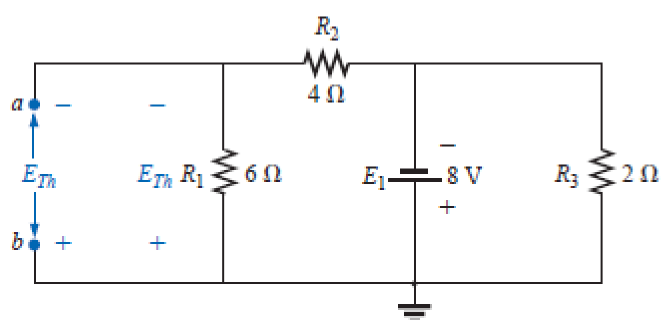


Solution

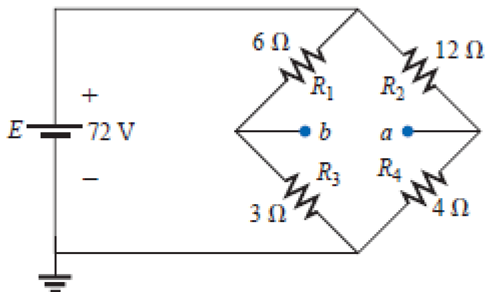
$$R_{Th} = R_1 \parallel R_2 = \frac{(6\ \Omega)(4\ \Omega)}{6\ \Omega + 4\ \Omega} = \frac{24\ \Omega}{10} = \mathbf{2.4\ \Omega}$$



$$E_{Th} = \frac{R_1E_1}{R_1 + R_2} = \frac{(6\ \Omega)(8\ \text{V})}{6\ \Omega + 4\ \Omega} = \frac{48\ \text{V}}{10} = \mathbf{4.8\ V}$$



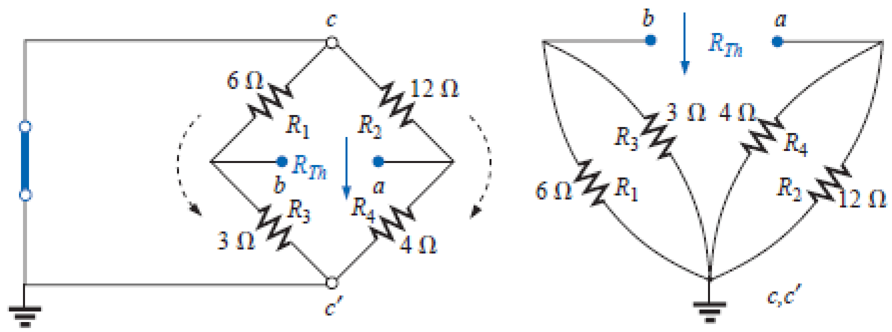
Example 3.3 Find the Thévenin equivalent circuit for the bridge network of Fig. below ?



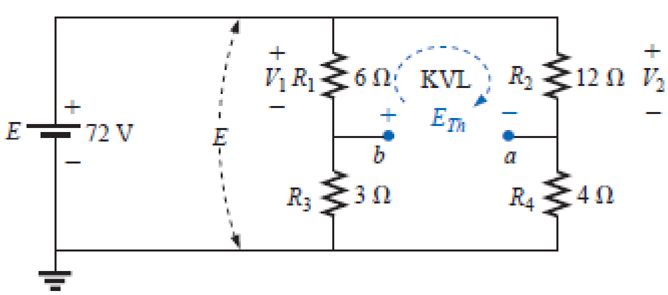
Solution :

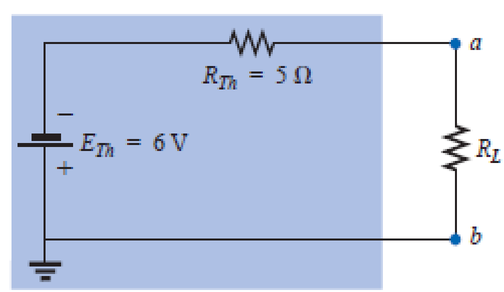
At first we find R_{th}

$$R_{Th} = R_{a\text{-}b} = R_1 \parallel R_3 + R_2 \parallel R_4$$
$$= 6\ \Omega \parallel 3\ \Omega + 4\ \Omega \parallel 12\ \Omega$$
$$= 2\ \Omega + 3\ \Omega = \mathbf{5\ \Omega}$$



(a) (b)





Then we find V_{TH}

$$V_1 = \frac{R_1 E}{R_1 + R_3} = \frac{(6\ \Omega)(72\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{432\ \text{V}}{9} = 48\ \text{V}$$

$$V_2 = \frac{R_2 E}{R_2 + R_4} = \frac{(12\ \Omega)(72\ \text{V})}{12\ \Omega + 4\ \Omega} = \frac{864\ \text{V}}{16} = 54\ \text{V}$$

Assuming the polarity shown for E_{Th} and applying Kirchhoff's voltage law to the top loop in the clockwise direction will result in

$$\Sigma_{\circlearrowright} V = +E_{Th} + V_1 - V_2 = 0$$

and $$E_{Th} = V_2 - V_1 = 54\text{ V} - 48\text{ V} = \mathbf{6\text{ V}}$$