3.1 The′venin’s Theorem

The′venin’s Theorem states the following: ***Any two-terminal, linear bilateral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor, as shown below:***

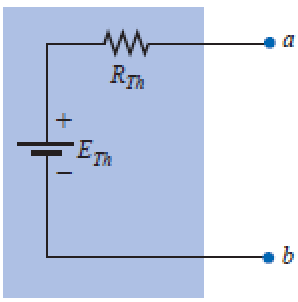


Fig.1 Thévenin equivalent circuit.

The following sequence of steps will lead to the proper value of R_{Th} and $E_{Th.}$ Preliminary:

R_{Th}

1. Remove that portion of the network across which the Thévenin equivalent circuit is to be found. In Fig.1, this requires that the load resistor RL be temporarily removed from the network.

2. Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.) .
3. Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

   E_{Th}:

4. Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that will lead to the most confusion and errors. In all cases, keep in mind that it is the open-circuit potential between the two terminals marked in step 2.) Conclusion:

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor RL between the terminals of the Thévenin equivalent circuit as shown in Fig. below.

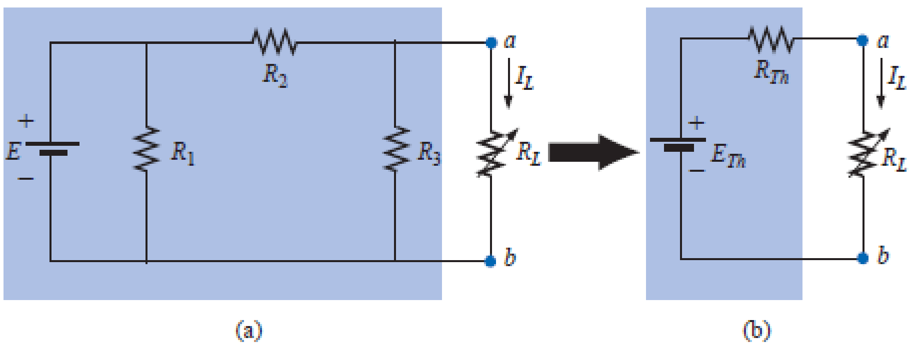
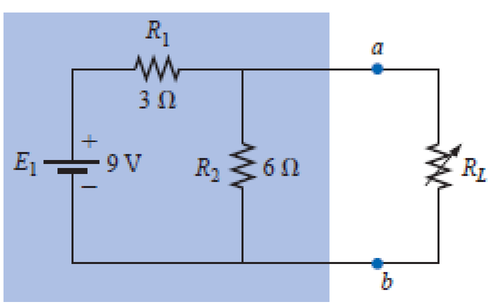


Fig.2 Substituting the Thévenin equivalent circuit for a complex network

Example 3.1 Find the Thévenin equivalent circuit for the network in the shaded area of the network of Fig. below. Then find the current through R_L for values of (2 ,10 ,and 100 Ω) **?**



Solution:

$$R_{Th} = R_1 \parallel R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \mathbf{2\ \Omega}$$

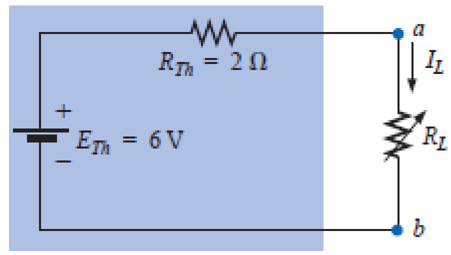
$$E_{Th} = \frac{R_2 E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = \mathbf{6\ V}$$

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

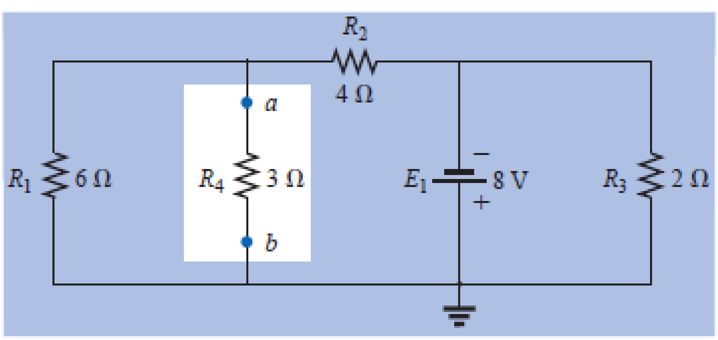
$R_L = 2\ \Omega$: $$I_L = \frac{6\ \text{V}}{2\ \Omega + 2\ \Omega} = \mathbf{1.5\ A}$$

$R_L = 10\ \Omega$: $$I_L = \frac{6\ \text{V}}{2\ \Omega + 10\ \Omega} = \mathbf{0.5\ A}$$

$R_L = 100\ \Omega$: $$I_L = \frac{6\ \text{V}}{2\ \Omega + 100\ \Omega} = \mathbf{0.059\ A}$$

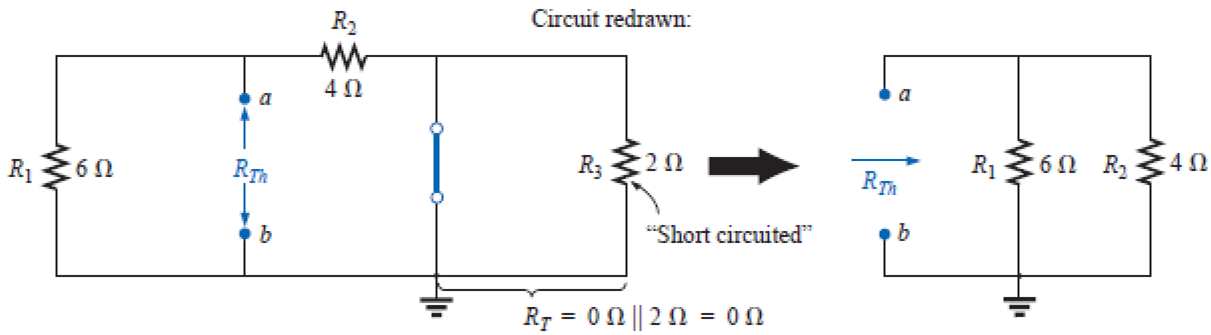


Example 3.2 Find the Thévenin equivalent circuit for the network in the shaded area of the network of Fig. below. ?

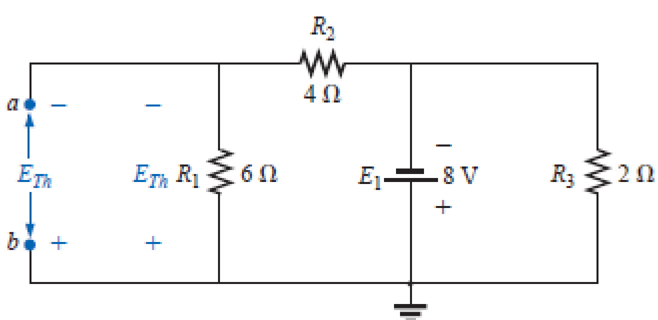


Solution

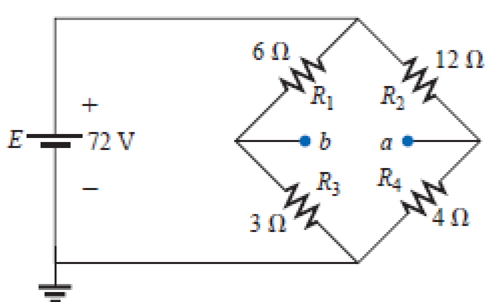
$$R_{Th} = R_1 \parallel R_2 = \frac{(6\ \Omega)(4\ \Omega)}{6\ \Omega + 4\ \Omega} = \frac{24\ \Omega}{10} = \mathbf{2.4\ \Omega}$$



$$E_{Th} = \frac{R_1 E_1}{R_1 + R_2} = \frac{(6\ \Omega)(8\ \text{V})}{6\ \Omega + 4\ \Omega} = \frac{48\ \text{V}}{10} = \mathbf{4.8\ V}$$



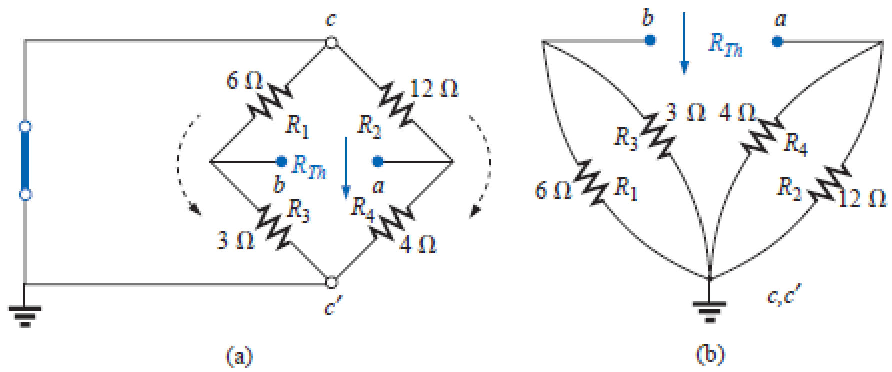
Example 3.3 Find the Thévenin equivalent circuit for the bridge network of Fig. below ?

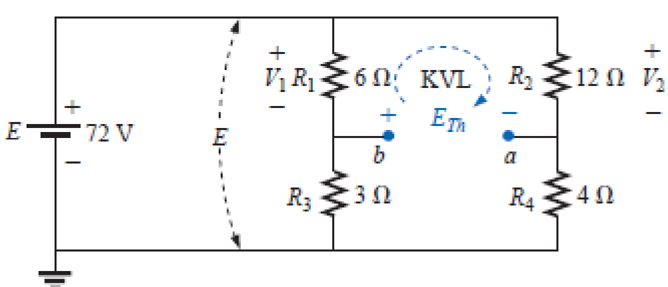


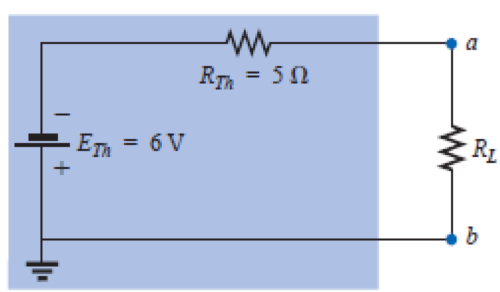
Solution :

At first we find R_{th}

$$\begin{aligned} R_{Th} = R_{a\text{-}b} &= R_1 \parallel R_3 + R_2 \parallel R_4 \\ &= 6\ \Omega \parallel 3\ \Omega + 4\ \Omega \parallel 12\ \Omega \\ &= 2\ \Omega + 3\ \Omega = \mathbf{5\ \Omega} \end{aligned}$$







Then we find V_{TH}

$$V_1 = \frac{R_1 E}{R_1 + R_3} = \frac{(6\ \Omega)(72\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{432\ \text{V}}{9} = 48\ \text{V}$$

$$V_2 = \frac{R_2 E}{R_2 + R_4} = \frac{(12\ \Omega)(72\ \text{V})}{12\ \Omega + 4\ \Omega} = \frac{864\ \text{V}}{16} = 54\ \text{V}$$

Assuming the polarity shown for E_{Th} and applying Kirchhoff's voltage law to the top loop in the clockwise direction will result in

$$\Sigma_{\circlearrowright} V = +E_{Th} + V_1 - V_2 = 0$$

and
$$E_{Th} = V_2 - V_1 = 54\text{ V} - 48\text{ V} = \mathbf{6\ V}$$

3.2 Norton's Theorem

Norton's theorem states the following:

An equivalent circuit consisting of a current source and a parallel resistor can replace any two-terminal linear bilateral dc network.

The discussion of Thévenin's theorem with respect to the equivalent circuit can also be applied to the Norton equivalent circuit. The steps leading to the proper values of I_N and R_N are now listed.

1. Remove that portion of the network across which the Norton equivalent circuit is found.
2. Mark the terminals of the remaining two-terminal network.

R_N:

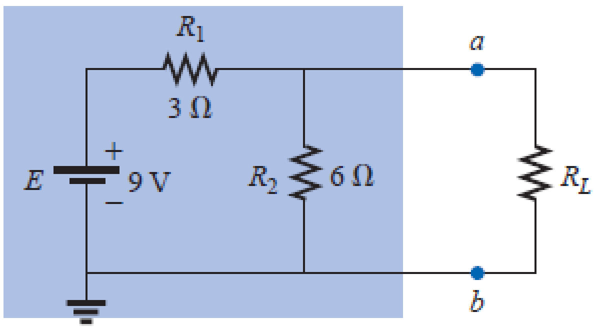
3. Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since $R_N = R_{Th}$, the procedure and value obtained using the approach described for Thévenin's theorem will determine the proper value of R_N.

I_N:

4. Calculate I_N by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.

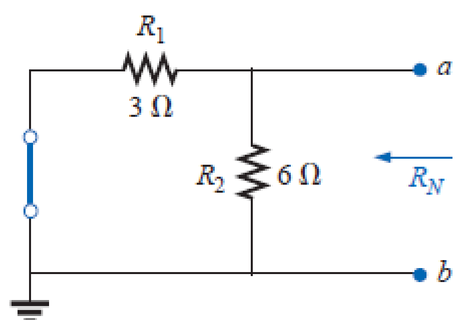
5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Example 3.4 Find the Norton equivalent circuit for the network in the shaded area of Fig. below ?



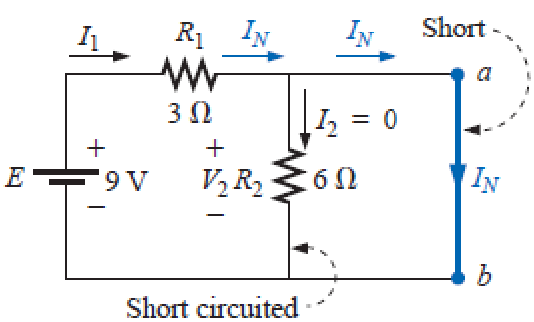
Solution:

1- R_N



$$R_N = R_1 \parallel R_2 = 3\ \Omega \parallel 6\ \Omega = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = \mathbf{2\ \Omega}$$

2- I_N

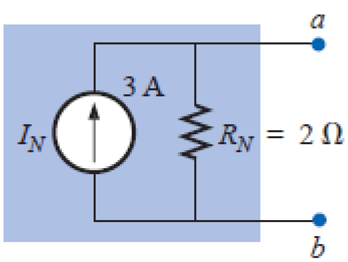


$$V_2 = I_2 R_2 = (0)6\ \Omega = 0\ \text{V}$$

Therefore,

$$I_N = \frac{E}{R_1} = \frac{9\ \text{V}}{3\ \Omega} = \mathbf{3\ A}$$

3- Equivalent Circuit

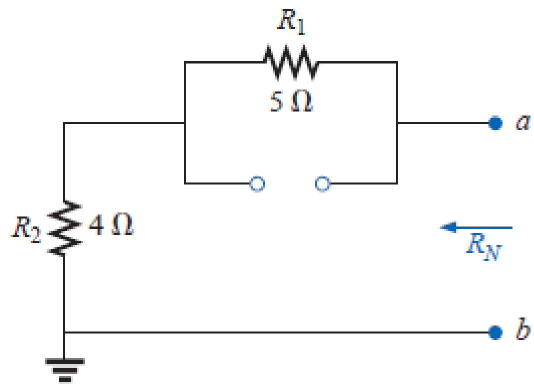


Example 3.5 Find the Norton equivalent circuit for the network external to the 9-Ω resistor in Fig. below ?

Solution

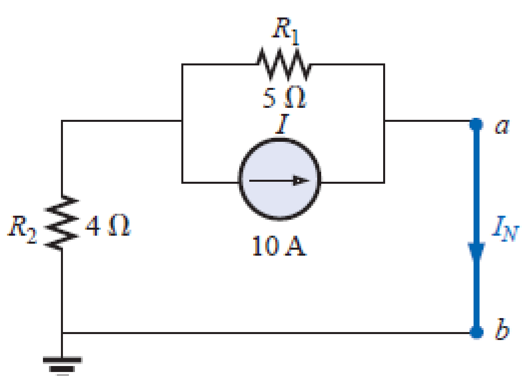
1- R_N

$$R_N = R_1 + R_2 = 5\ \Omega + 4\ \Omega = \mathbf{9\ \Omega}$$

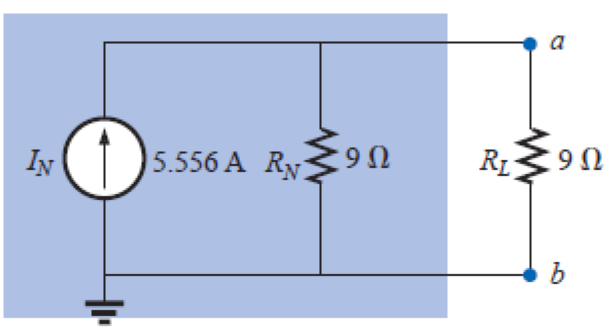


2- I_N

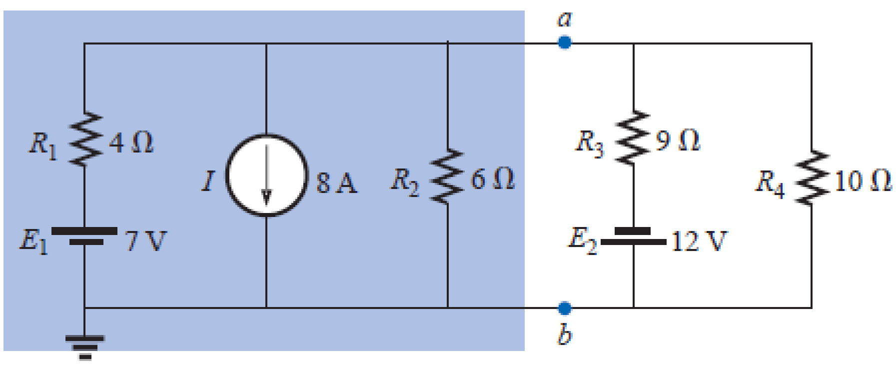
$$I_N = \frac{R_1 I}{R_1 + R_2} = \frac{(5\ \Omega)(10\ \text{A})}{5\ \Omega + 4\ \Omega} = \frac{50\ \text{A}}{9} = \mathbf{5.556\ A}$$



3- Equivalent Circuit



Example 3.6 (Two sources) Find the Norton equivalent circuit for the portion of the network to the left of *a-b* in Fig. below ??



Solution:

1- R_N

$$R_N = R_1 \parallel R_2 = 4\ \Omega \parallel 6\ \Omega = \frac{(4\ \Omega)(6\ \Omega)}{4\ \Omega + 6\ \Omega} = \frac{24\ \Omega}{10} = \mathbf{2.4\ \Omega}$$

2- I_N

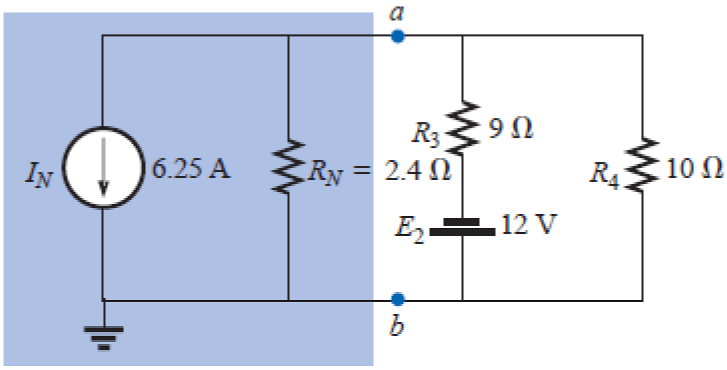
$$I'_N = \frac{E_1}{R_1} = \frac{7\text{ V}}{4\ \Omega} = 1.75\text{ A}$$

$$I''_N = I = 8\text{ A}$$

The result is

$$I_N = I''_N - I'_N = 8\text{ A} - 1.75\text{ A} = \mathbf{6.25\ A}$$

3- Equivalent Circuit



3.3 Current Sources

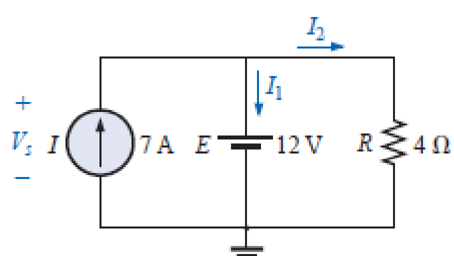
The current source is often referred to as the *dual* of the voltage source. A battery supplies a *fixed* voltage, and the source current can vary; but the current source supplies a *fixed* current to the branch in which it is located, while its terminal voltage may vary as determined by the network to which it is applied. Note from the above that *duality* simply implies an interchange of current and voltage to distinguish the characteristics of one source from the other.

A current source determines the current in the branch in which it is located

and

the magnitude and polarity of the voltage across a current source are a function of the network to which it is applied.

Example 3.7 Find the voltage *Vs* and the currents I_1 and I_2 ??



Solution:

$$V_s = E = \mathbf{12\ V}$$

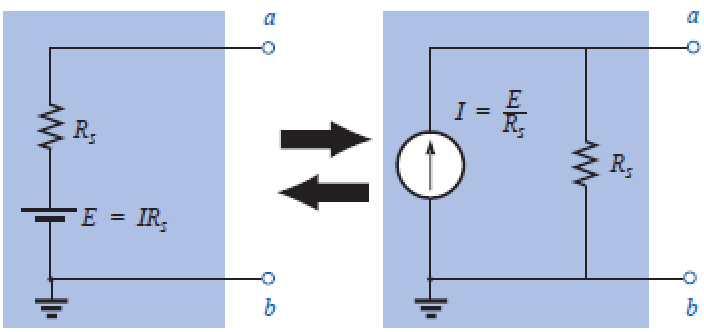
$$I_2 = \frac{V_R}{R} = \frac{E}{R} = \frac{12\text{ V}}{4\ \Omega} = \mathbf{3\ A}$$

Applying Kirchhoff's current law:

$$I = I_1 + I_2$$

and $$I_1 = I - I_2 = 7\text{ A} - 3\text{ A} = \mathbf{4\ A}$$

3.4 Source Conversions



Example 3.8

a. Convert the voltage source of Fig.(a) below to a current source, and calculate the current through the 4-Ω load for each source.
b. Replace the 4-Ω load with a 1-kΩ load, and calculate the current I_L for the voltage source.

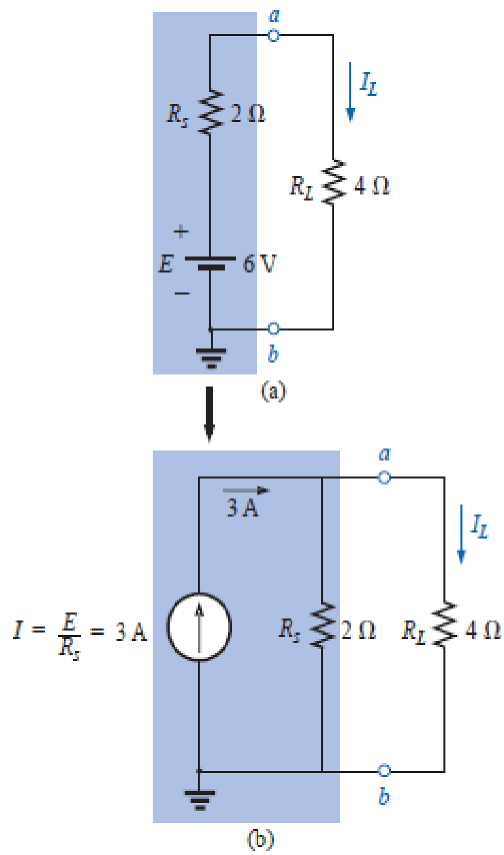
Solutions:

a.

$$I_L = \frac{E}{R_s + R_L} = \frac{6\ \text{V}}{2\ \Omega + 4\ \Omega} = \mathbf{1\ A}$$

$$I_L = \frac{R_s I}{R_s + R_L} = \frac{(2\ \Omega)(3\ \text{A})}{2\ \Omega + 4\ \Omega} = \mathbf{1\ A}$$

b. $I_L = \dfrac{E}{R_s + R_L} = \dfrac{6\ \text{V}}{2\ \Omega + 1\ \text{k}\Omega} \cong \mathbf{5.99\ mA}$



Example 3.9 Reduce the network of Fig. below to a single current source, and calculate the current through R_L ??

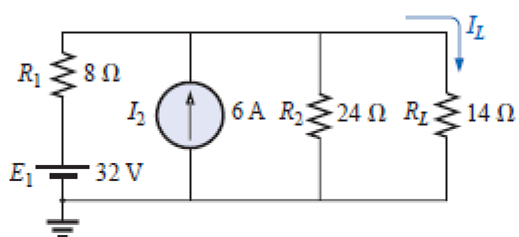
Solution

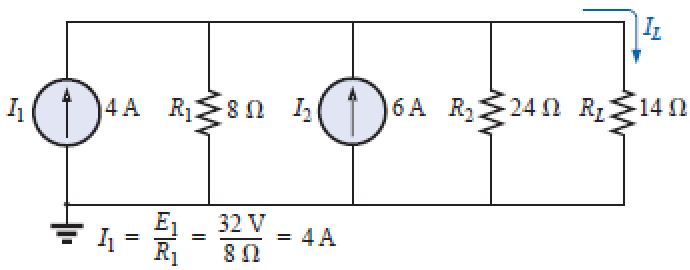
$$I_s = I_1 + I_2 = 4\ \text{A} + 6\ \text{A} = \mathbf{10\ A}$$

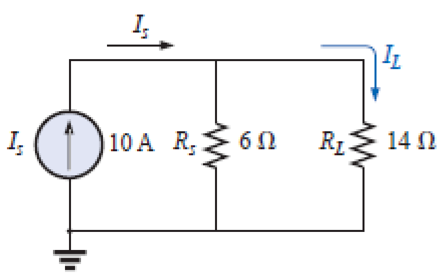
and $$R_s = R_1 \parallel R_2 = 8\ \Omega \parallel 24\ \Omega = \mathbf{6\ \Omega}$$

Applying the current divider rule

$$I_L = \frac{R_s I_s}{R_s + R_L} = \frac{(6\ \Omega)(10\ \text{A})}{6\ \Omega + 14\ \Omega} = \frac{60\ \text{A}}{20} = \mathbf{3\ A}$$







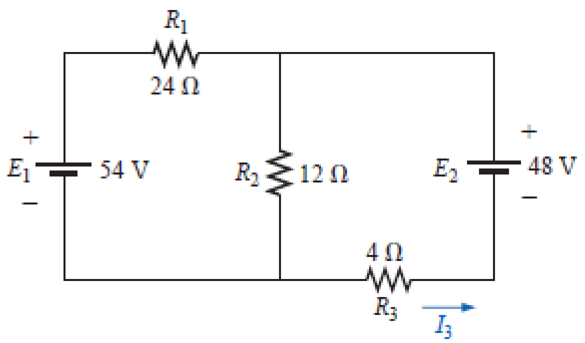
3.5 Superposition Theorem

The **superposition theorem,** like the methods of the last chapter, can be used to find the solution to networks with two or more sources that are not in series or parallel. The most obvious advantage of this method is that it does not require the use of a mathematical technique such as determinants to find the required voltages or currents. Instead, each source is treated independently, and the algebraic sum is found to determine a particular unknown quantity of the network.

The superposition theorem states the following:

The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.

Example 3.10 Using superposition, determine the current through the 4-Ω resistor ??



Solution: *Considering the effects of a 54-V source*

$$R_T = R_1 + R_2 \parallel R_3 = 24\ \Omega + 12\ \Omega \parallel 4\ \Omega = 24\ \Omega + 3\ \Omega = 27\ \Omega$$

$$I = \frac{E_1}{R_T} = \frac{54\ \text{V}}{27\ \Omega} = 2\ \text{A}$$

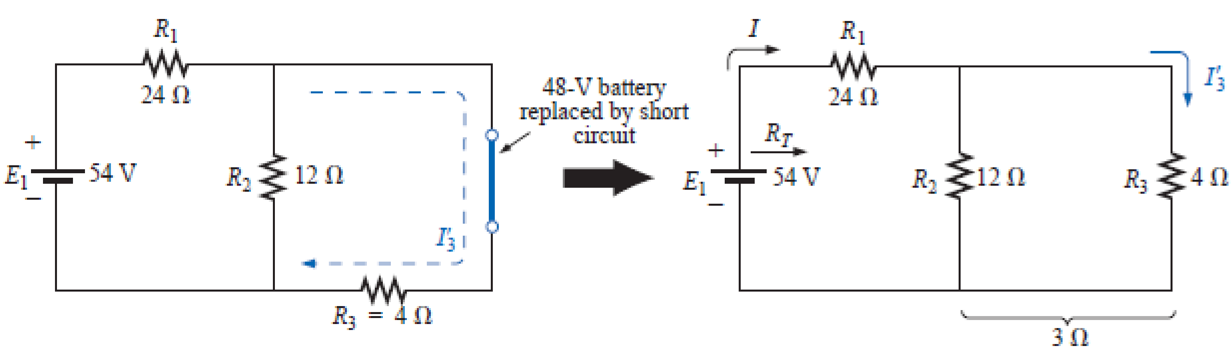
Using the current divider rule,

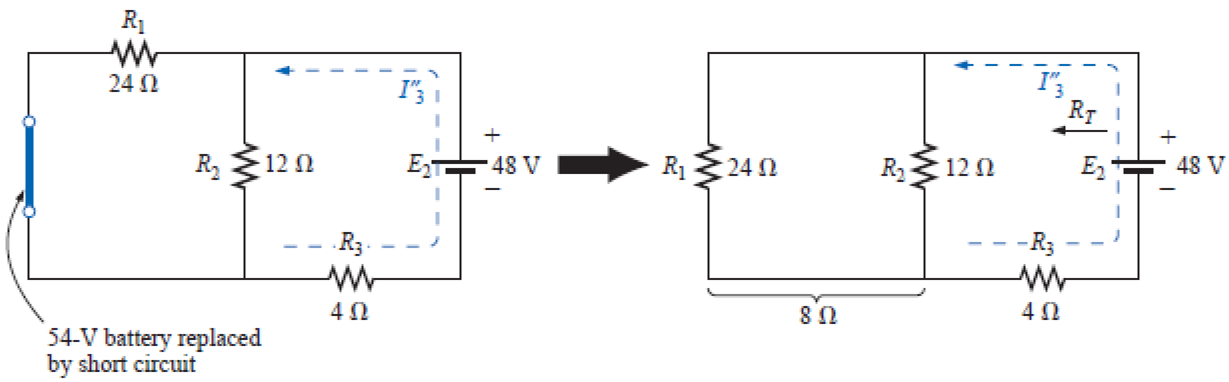
$$I'_3 = \frac{R_2 I}{R_2 + R_3} = \frac{(12\ \Omega)(2\ \text{A})}{12\ \Omega + 4\ \Omega} = \frac{24\ \text{A}}{16} = 1.5\ \text{A}$$

Considering the effects of the 48-V source

$$R_T = R_3 + R_1 \parallel R_2 = 4\ \Omega + 24\ \Omega \parallel 12\ \Omega = 4\ \Omega + 8\ \Omega = 12\ \Omega$$

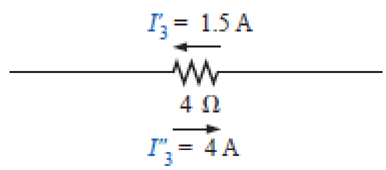
$$I''_3 = \frac{E_2}{R_T} = \frac{48\ \text{V}}{12\ \Omega} = 4\ \text{A}$$





The total current through the 4-Ω resistor

$$I_3 = I''_3 - I'_3 = 4\text{ A} - 1.5\text{ A} = \mathbf{2.5\ A} \qquad (\text{direction of } I''_3)$$



3.6 Maximum Power Transfer Theorem

The **maximum power transfer theorem** states the following:

A load will receive maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thévenin resistance of the network as "seen" by the load.

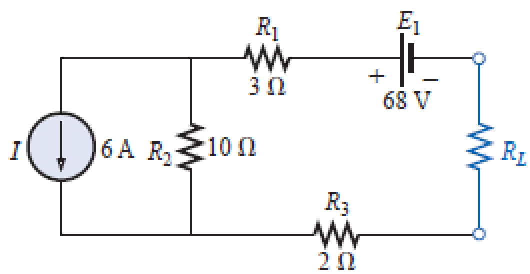
For the **Thévenin** circuit

$$\boxed{P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}}} \qquad \text{(watts, W)}$$

For the Norton circuit

$$\boxed{P_{L_{max}} = \frac{I_N^2 R_N}{4}} \qquad \text{(W)}$$

Example 3.11 Find the value of R_L in Fig. below for maximum power to R_L, and determine the maximum power ??



Solution

$$R_{Th} = R_1 + R_2 + R_3 = 3\ \Omega + 10\ \Omega + 2\ \Omega = 15\ \Omega$$

and $$R_L = R_{Th} = \mathbf{15\ \Omega}$$

$$V_1 = V_3 = 0\ \mathrm{V}$$

and $$V_2 = I_2R_2 = IR_2 = (6\ \mathrm{A})(10\ \Omega) = 60\ \mathrm{V}$$

Applying Kirchhoff's voltage law,

$$\Sigma_{\circlearrowright} V = -V_2 - E_1 + E_{Th} = 0$$

and $$E_{Th} = V_2 + E_1 = 60\ \mathrm{V} + 68\ \mathrm{V} = 128\ \mathrm{V}$$

Thus, $$P_{L_{\max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(128\ \mathrm{V})^2}{4(15\ \Omega)} = \mathbf{273.07\ W}$$