Chapter 2
Series and parallel resistance

Introduction

Chapter 1 introduced basic concepts such as current, voltage, and power in an electric circuit. To actually determine the values of these variables in a given circuit requires that we understand some fundamental laws that govern electric circuits. These laws, known as Ohm's law and Kirchhoff's laws, form the foundation upon which electric circuit analysis is built. In this chapter, in addition to these laws, we shall discuss some techniques commonly applied in circuit design and analysis. These techniques include combining resistors in series or parallel, voltage division, current division, and delta-to-wye and wye-to-delta transformations.

2.1 Ohm's Law

Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as *Ohm's law*. Ohm's law states that the voltage *v* across a resistor is directly proportional to the current *i* flowing through the resistor.

That is,

$$v \propto i \qquad \textbf{2.1}$$

Ohm defined the constant of proportionality for a resistor to be the resistance, *R*. (The resistance is a material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.) Thus, Eq. (2.1) becomes

$$\boxed{v = iR} \qquad \textbf{2.2}$$

Ohm defined the constant of proportionality for a resistor to be the resistance, *R*. (The resistance is a material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.) Thus, Eq. (2.2) becomes which is the mathematical form of Ohm's law. *R* in Eq. (2.3) is measured in the unit of ohms, designated . Thus, The *resistance R* of an element denotes its ability to resist the flow of electric current; it is measured in ohms (**Ω**).

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A short circuit is a circuit element with resistance approaching zero.

An open circuit is a circuit element with resistance approaching infinity.

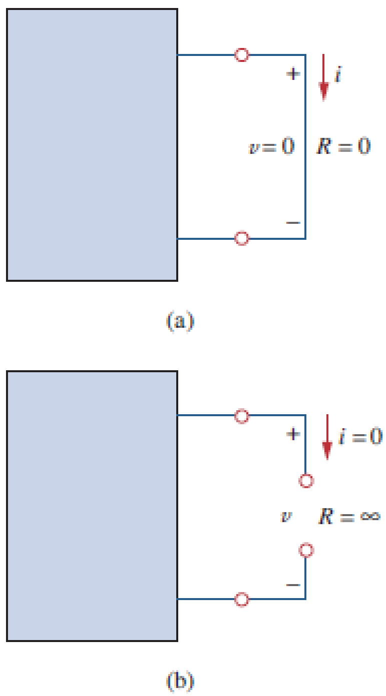

Figure 2.1
(a) Short circuit (R=0) , (b) Open circuit ($R = \infty$).

A useful quantity in circuit analysis is the reciprocal of resistance R, known as *conductance* and denoted by G:

$$\boxed{G = \frac{1}{R} = \frac{i}{v}} \qquad \textbf{(2.3)}$$

Conductance is the ability of an element to conduct electric current; it is measured in mhos (℧) or siemens (S).

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$$i = Gv \qquad \textbf{(2.4)}$$

The power dissipated by a resistor can be expressed in terms of *R*.

$$p = vi = i^2R = \frac{v^2}{R} \quad \textbf{watts} \quad \textbf{(2.5)}$$

The power dissipated by a resistor may also be expressed in terms of G as

$$p = vi = v^2G = \frac{i^2}{G} \quad \textbf{watts} \quad \textbf{(2.6)}$$

The Power delivered from the source is

$$\boxed{P = EI} \qquad \text{(watts)} \qquad \textbf{(2.7)}$$

with *E* the battery terminal voltage and *I* the current through the source.

EXAMPLE 2.1 : Determine the current through a 5-kΩ resistor when the power dissipated by the element is 20 mW.

Solution:

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{20 \times 10^{-3}\,\text{W}}{5 \times 10^{3}\,\Omega}} = \sqrt{4 \times 10^{-6}} = 2 \times 10^{-3}\,\text{A}$$
$$= \mathbf{2\ mA}$$

2.2 Series Circuits

A **circuit** consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow. The circuit of Fig. 2.2(a) has three elements joined at three terminal points (*a*, *b*, and *c*) to provide a closed path for the current *I*.

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Two elements are in series if

1. They have only one terminal in common (i.e., one lead of one is connected to only one lead of the other).
2. The common point between the two elements is not connected to another current-carrying element.

(a) series circuit **(b) R1 and R2 not in series**

Figure 2.2

In general, to find the total resistance of N resistors in series, the following equation is applied:

$$\boxed{R_T = R_1 + R_2 + R_3 + \cdots + R_N} \qquad \text{(ohms, } \Omega) \qquad \textbf{(2.8)}$$

The current drawn from the source can be determined using Ohm’s law, as follows:

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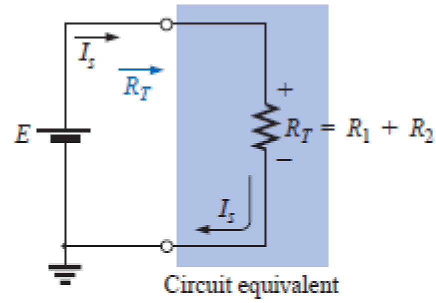
$$\boxed{I_s = \frac{E}{R_T}} \quad \text{(amperes, A)} \qquad \textbf{(2.9)}$$

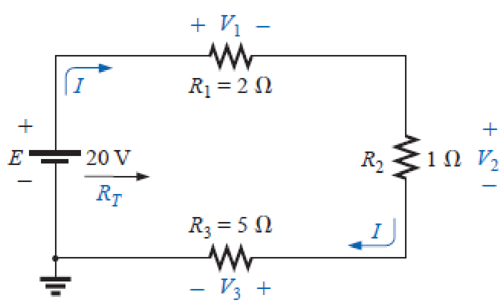


Circuit equivalent

The fact that the current is the same through each element of Fig. 2.2 (a) permits a direct calculation of the voltage across each resistor using Ohm's law; that is,

$$\boxed{V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3,\ \ldots,\ V_N = IR_N} \quad \text{(volts, V)} \qquad \textbf{(2.10)}$$

EXAMPLE 2.2



a. Find the total resistance for the series circuit
b. Calculate the source current I_s.
c. Determine the voltages V_1, V_2, and V_3.
d. Calculate the power dissipated by R_1, R_2, and R_3.
e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).

Solutions:

a. $R_T = R_1 + R_2 + R_3 = 2\ \Omega + 1\ \Omega + 5\ \Omega = \mathbf{8\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\ \text{V}}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = IR_1 = (2.5\ \text{A})(2\ \Omega) = \mathbf{5\ V}$
$V_2 = IR_2 = (2.5\ \text{A})(1\ \Omega) = \mathbf{2.5\ V}$
$V_3 = IR_3 = (2.5\ \text{A})(5\ \Omega) = \mathbf{12.5\ V}$

d. $P_1 = V_1 I_1 = (5\ \text{V})(2.5\ \text{A}) = \mathbf{12.5\ W}$
$P_2 = I_2^2 R_2 = (2.5\ \text{A})^2(1\ \Omega) = \mathbf{6.25\ W}$
$P_3 = V_3^2/R_3 = (12.5\ \text{V})^2/5\ \Omega = \mathbf{31.25\ W}$

e. $P_{del} = EI = (20\ \text{V})(2.5\ \text{A}) = \mathbf{50\ W}$
$P_{del} = P_1 + P_2 + P_3$
$50\ \text{W} = 12.5\ \text{W} + 6.25\ \text{W} + 31.25\ \text{W}$
$\underline{50\ \text{W} = 50\ \text{W} \quad \text{(checks)}}$

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2.3 Parallel Circuits

Two network configurations, series and parallel, form the framework for some of the most complex network structures. A clear understanding of each will pay enormous dividends as more complex methods and networks are examined. We will now examine the **parallel circuit** and all the methods and laws associated with this important configuration.

Two elements, branches, or networks are in parallel if they have two points in common.

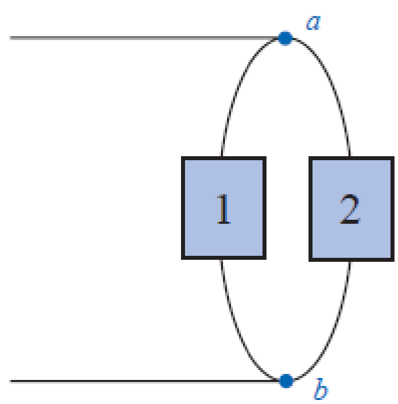

Figure 2.3 ***Parallel elements.***

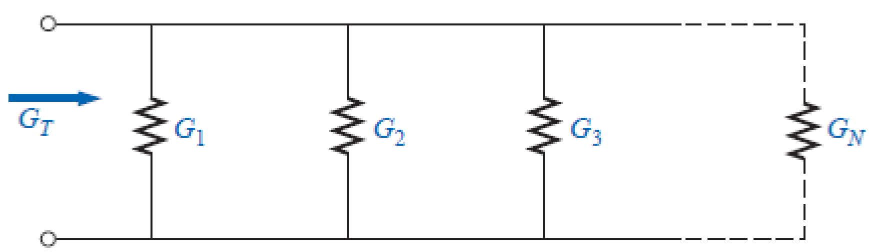

Figure 2.4 ***Determining the total conductance of parallel conductances.***

For parallel elements, the total conductance is the sum of the individual conductances.

$$\boxed{G_T = G_1 + G_2 + G_3 + \cdots + G_N} \qquad (2.11)$$

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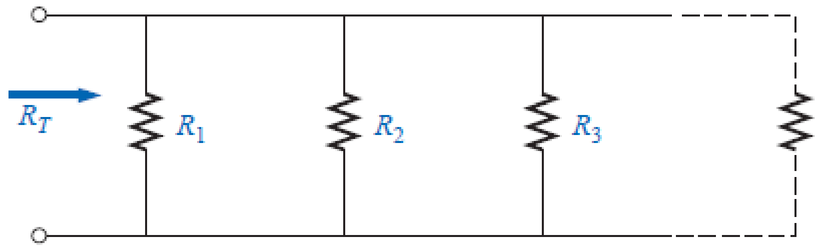
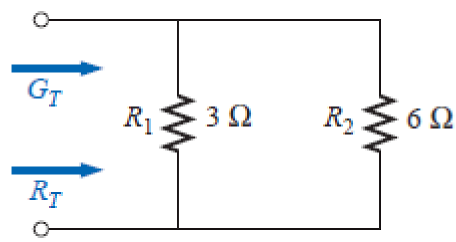


Figure 2.5 ***Determining the total resistance of parallel resistors***.

$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}} \qquad \textbf{(2.12)}$$

EXAMPLE 2.3 Determine the total conductance and resistance for the parallel network of Fig. 2.5.



solution

$$G_T = G_1 + G_2 = \frac{1}{3\ \Omega} + \frac{1}{6\ \Omega} = 0.333\ \text{S} + 0.167\ \text{S} = \mathbf{0.5\ S}$$

and
$$R_T = \frac{1}{G_T} = \frac{1}{0.5\ \text{S}} = \mathbf{2\ \Omega}$$

the total resistance of two parallel resistors is the product of the two divided by their sum.

$$\boxed{R_T = \frac{R_1 R_2}{R_1 + R_2}} \qquad \textbf{(2.13)}$$

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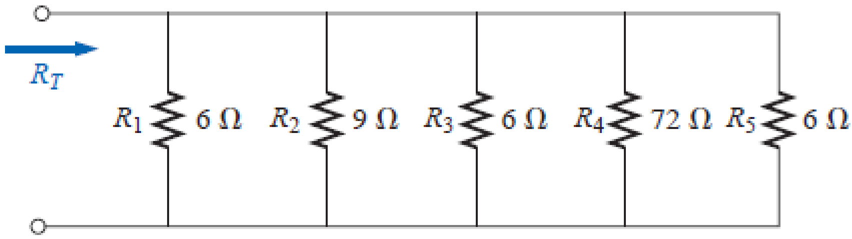
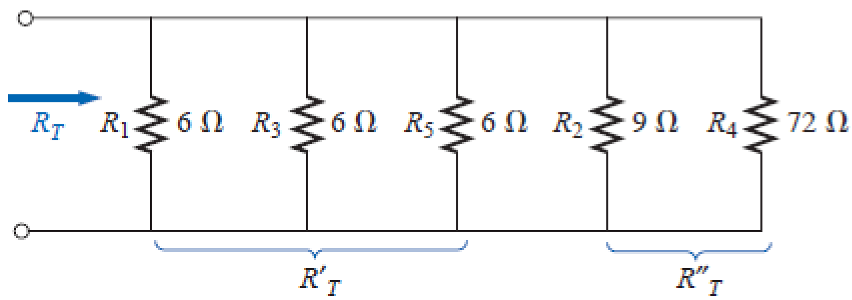
EXAMPLE 2.4 Calculate the total resistance of the parallel network of Fig. below ??



Solution: The network is redrawn in Fig. below :



$$R'_T = \frac{R}{N} = \frac{6\ \Omega}{3} = 2\ \Omega$$

$$R''_T = \frac{R_2 R_4}{R_2 + R_4} = \frac{(9\ \Omega)(72\ \Omega)}{9\ \Omega + 72\ \Omega} = \frac{648\ \Omega}{81} = 8\ \Omega$$

and $$R_T = \underbrace{R'_T \parallel R''_T}_{\text{In parallel with}}$$

$$= \frac{R'_T R''_T}{R'_T + R''_T} = \frac{(2\ \Omega)(8\ \Omega)}{2\ \Omega + 8\ \Omega} = \frac{16\ \Omega}{10} = \mathbf{1.6\ \Omega}$$

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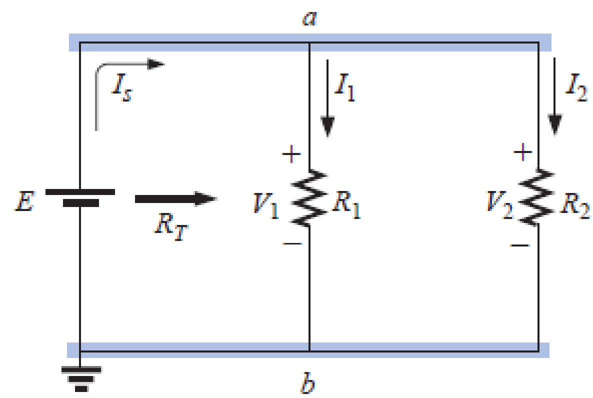
2.4 PARALLEL CIRCUITS

The network of Fig. 6.21 is the simplest of parallel circuits. All the elements have terminals *a* and *b* in common. The total resistance is determined by $R_T = R_1R_2/(R_1 + R_2)$, and the source current by $I_s = E/R_T$. Throughout the text, the subscript *s* will be used to denote a property of the source. Since the terminals of the battery are connected directly across the resistors R_1 and R_2, the following should be obvious:



The voltage across parallel elements is the same. Using this fact will result in

$$V_1 = V_2 = E$$

and
$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1}$$

with
$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}$$

If we take the equation for the total resistance and multiply both sides by the applied voltage, we obtain

$$E\left(\frac{1}{R_T}\right) = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

and
$$\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2}$$

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Substituting the Ohm's law relationships appearing above, we find that the source current

$$I_s = I_1 + I_2$$

permitting the following conclusion:

For single-source parallel networks, the source current (I_s) is equal to the sum of the individual branch currents.

The power dissipated by the resistors and delivered by the source can be determined from

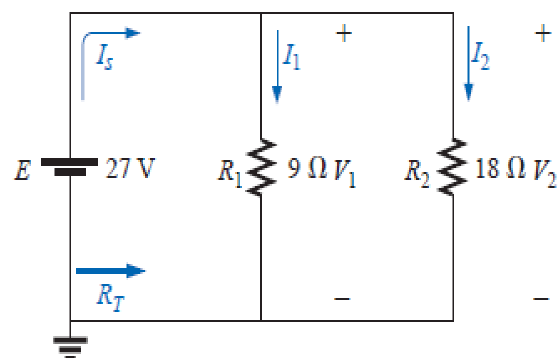
$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}$$

$$P_2 = V_2 I_2 = I_2^2 R_2 = \frac{V_2^2}{R_2}$$

$$P_s = E I_s = I_s^2 R_T = \frac{E^2}{R_T}$$

Example:

a. Calculate R_T.
b. Determine I_s.
c. Calculate I_1 and I_2, and demonstrate that $I_s = I_1 + I_2$.
d. Determine the power to each resistive load.
e. Determine the power delivered by the source, and compare it to the total power dissipated by the resistive elements.



Solutions:

a. $R_T = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{(9\ \Omega)(18\ \Omega)}{9\ \Omega + 18\ \Omega} = \dfrac{162\ \Omega}{27} = \mathbf{6\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{27\ \text{V}}{6\ \Omega} = \mathbf{4.5\ A}$

c. $I_1 = \dfrac{V_1}{R_1} = \dfrac{E}{R_1} = \dfrac{27\text{ V}}{9\ \Omega} = \mathbf{3\ A}$

$I_2 = \dfrac{V_2}{R_2} = \dfrac{E}{R_2} = \dfrac{27\text{ V}}{18\ \Omega} = \mathbf{1.5\ A}$

$I_s = I_1 + I_2$

$4.5\text{ A} = 3\text{ A} + 1.5\text{ A}$

$\mathbf{4.5\ A = 4.5\ A}$ (checks)

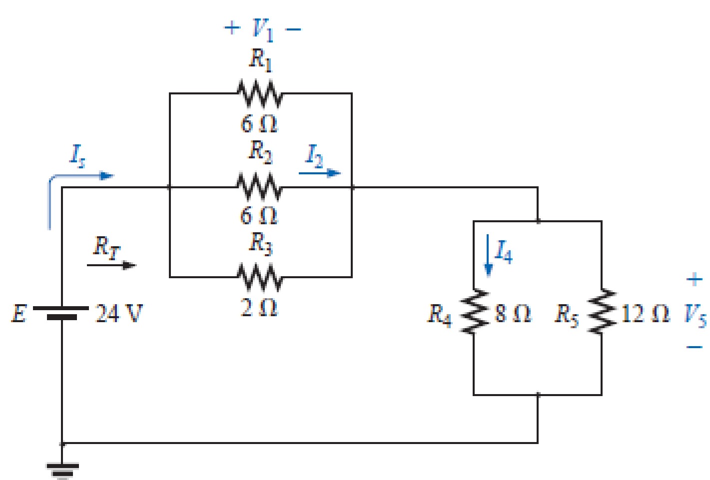
d. $P_1 = V_1I_1 = EI_1 = (27\text{ V})(3\text{ A}) = \mathbf{81\ W}$
$P_2 = V_2I_2 = EI_2 = (27\text{ V})(1.5\text{ A}) = \mathbf{40.5\ W}$

e. $P_s = EI_s = (27\text{ V})(4.5\text{ A}) = \mathbf{121.5\ W}$
$= P_1 + P_2 = 81\text{ W} + 40.5\text{ W} = \mathbf{121.5\ W}$

2.5 Series-Parallel Networks

series-parallel networks ***are networks that contain both series and parallel circuit configurations.***

EXAMPLE 2.6 Find the indicated currents and voltages for the network of Fig below ??



Solution:

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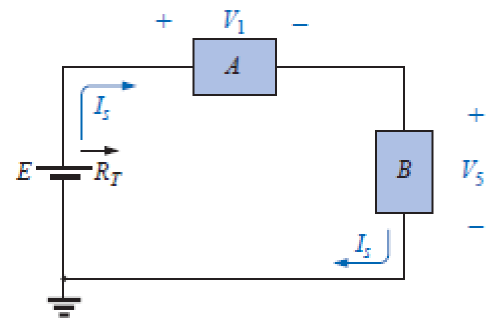


$$R_{1\|2} = \frac{R}{N} = \frac{6\ \Omega}{2} = 3\ \Omega$$

$$R_A = R_{1\|2\|3} = \frac{(3\ \Omega)(2\ \Omega)}{3\ \Omega + 2\ \Omega} = \frac{6\ \Omega}{5} = 1.2\ \Omega$$

$$R_B = R_{4\|5} = \frac{(8\ \Omega)(12\ \Omega)}{8\ \Omega + 12\ \Omega} = \frac{96\ \Omega}{20} = 4.8\ \Omega$$

$$R_T = R_{1\|2\|3} + R_{4\|5} = 1.2\ \Omega + 4.8\ \Omega = \mathbf{6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{24\ \text{V}}{6\ \Omega} = \mathbf{4\ A}$$

with

$$V_1 = I_s R_{1\|2\|3} = (4\ \text{A})(1.2\ \Omega) = \mathbf{4.8\ V}$$

$$V_5 = I_s R_{4\|5} = (4\ \text{A})(4.8\ \Omega) = \mathbf{19.2\ V}$$

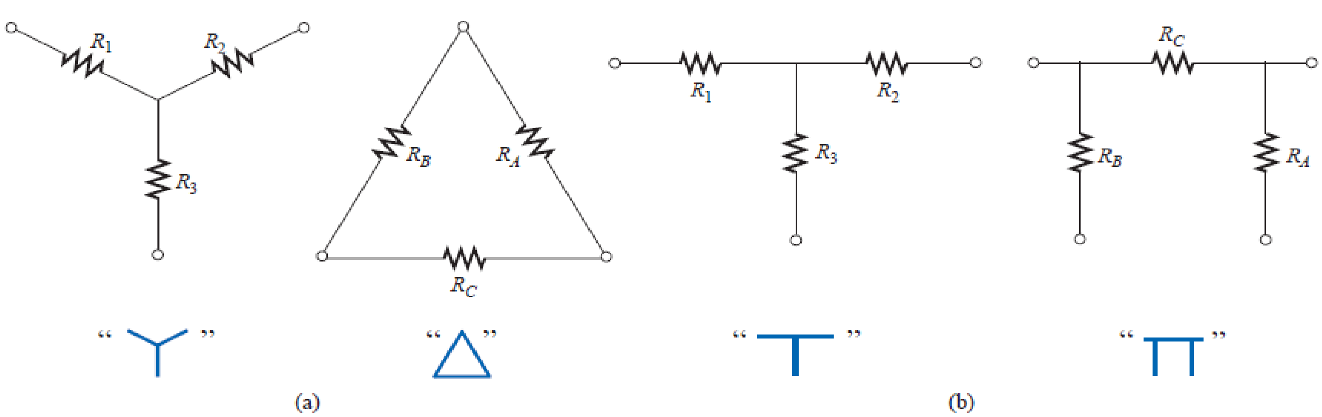
Applying Ohm's law,

$$I_4 = \frac{V_5}{R_4} = \frac{19.2\ \text{V}}{8\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_1}{R_2} = \frac{4.8\ \text{V}}{6\ \Omega} = \mathbf{0.8\ A}$$

2.6 Y-D (T-p) AND D-Y (p-T) CONVERSIONS

Circuit configurations are often encountered in which the resistors do not appear to be in series or parallel.



The Y (T) and D (p) *configurations.*

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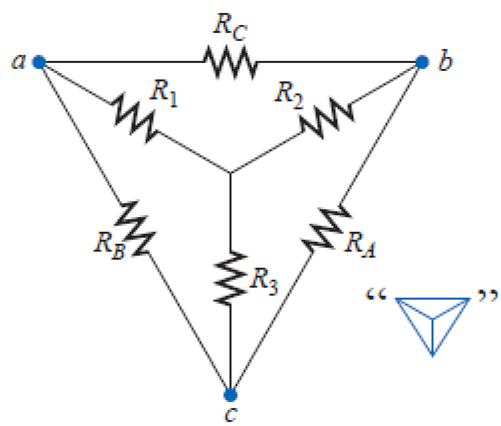
Under these conditions, it may be necessary to convert the circuit from one form to another to solve for any unknown quantities if mesh or nodal analysis is not applied. versa. Two circuit configurations that often account for these difficulties are the **wye** and **delta configurations**. They are also referred to as the **tee (T)** and **pi (π),** respectively,. Note that the pi is actually an inverted delta. The purpose of this section is to develop the equations for converting from D to Y, or vice



Introducing the concept of D-*Y or Y*-D *conversions.*

1. To obtain the relationships necessary to convert from a D to a Y

resulting in the following expression for R_3 in terms of R_A, R_B, and R_C:

$$\boxed{R_3 = \frac{R_A R_B}{R_A + R_B + R_C}} \qquad \textbf{(2.14)}$$

Following the same procedure for R_1 and R_2, we have

$$\boxed{R_1 = \frac{R_B R_C}{R_A + R_B + R_C}} \qquad \textbf{(2.15)}$$

and

$$\boxed{R_2 = \frac{R_A R_C}{R_A + R_B + R_C}} \qquad \textbf{(2.16)}$$

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2. To obtain the relationships necessary to convert from a Y to a D

and $$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3} \quad \textbf{(2.17)}$$

We follow the same procedure for R_B and R_A:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1} \quad \textbf{(2.18)}$$

and $$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2} \quad \textbf{(2.19)}$$

3. If R1 = R2 =R3 or RA = RB =RC

$$R_Y = \frac{R_\Delta}{3}$$

or $$R_\Delta = 3R_Y$$

EXAMPLE 2.7 Convert the D of Fig. below to a Y ??

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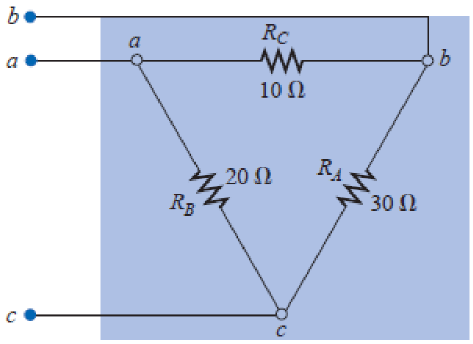
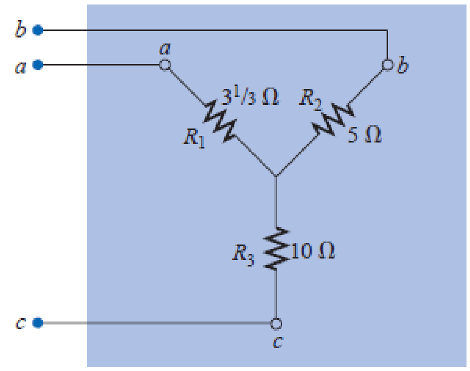




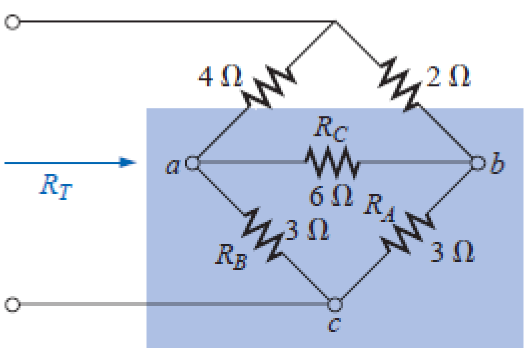
Solution:

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = \mathbf{3\tfrac{1}{3}\ \Omega}$$

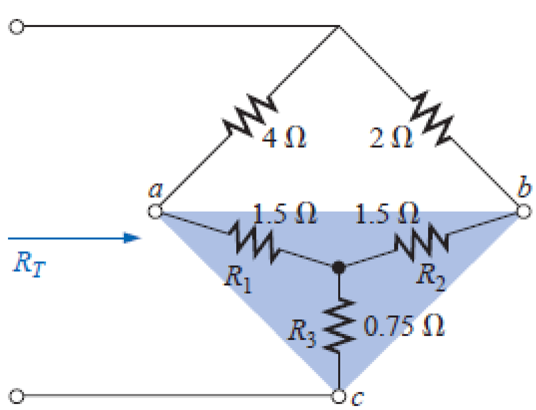
$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = \mathbf{10\ \Omega}$$

EXAMPLE 2.8 Find the total resistor of figure below ??



“▽”



Solution:

Two resistors of the Δ were equal; therefore, two resistors of the Y will be equal.

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

$$R_T = 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)}$$

$$= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega}$$

$$= 0.75\ \Omega + 2.139\ \Omega$$

$$R_T = \mathbf{2.889\ \Omega}$$

2.7 KIRCHHOFF’S VOLTAGE LAW

Kirchhoff’s voltage law ***(KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.*** A **closed loop** is any continuous path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit.

Example 2.9 Using Kirchhoff’s voltage law, determine the unknown voltages for the network of Fig. below .

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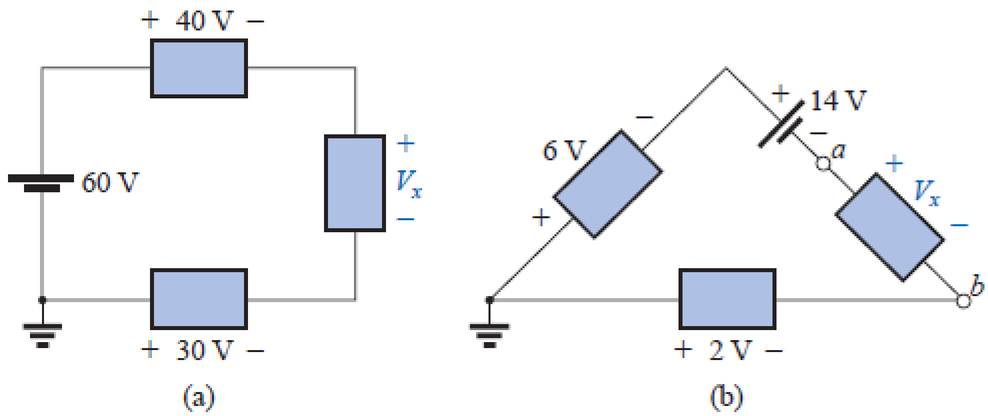


Solution:

For Fig a

$$60\text{ V} - 40\text{ V} - V_x + 30\text{ V} = 0$$

and

$$V_x = 60\text{ V} + 30\text{ V} - 40\text{ V} = 90\text{ V} - 40\text{ V}$$
$$= \mathbf{50\ V}$$

For Fig b

$$-6\text{ V} - 14\text{ V} - V_x + 2\text{ V} = 0$$

and

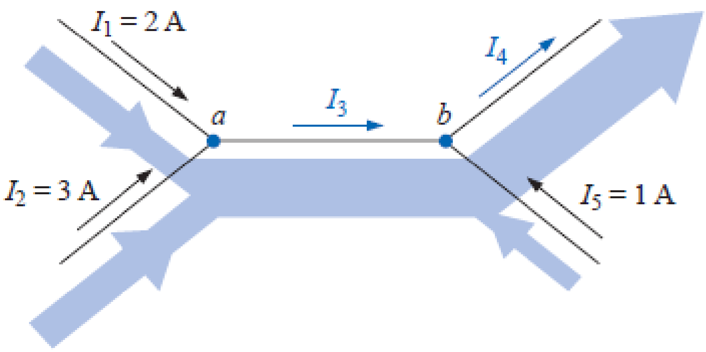
$$V_x = -20\text{ V} + 2\text{ V}$$
$$= \mathbf{-18\ V}$$

2.8 KIRCHHOFF'S CURRENT LAW

the sum of the currents entering an area, system, or junction must equal the sum of the currents leaving the area, system, or junction.

$$\boxed{\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}} \qquad \textbf{(2.14)}$$

Example 2.10 Determine the currents I_3 and I_4 of Fig. below using Kirchhoff's current law ??



Solution:

At a:

$$\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$$
$$I_1 + I_2 = I_3$$
$$2\text{ A} + 3\text{ A} = I_3$$
$$I_3 = \mathbf{5\ A}$$

At b:

$$\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$$
$$I_3 + I_5 = I_4$$
$$5\text{ A} + 1\text{ A} = I_4$$
$$I_4 = \mathbf{6\ A}$$

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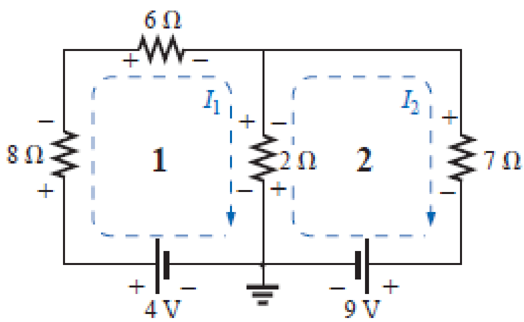
2.9 Mesh Analyses

The mesh-analysis approach simply eliminates the need to substitute the results of Kirchhoff's current law into the equations derived from Kirchhoff's voltage law. It is now accomplished in the initial writing of the equations.

EXAMPLE 2.10 Write the mesh equations for the network of Fig. below, and find the current through the 7-Ω resistor ??



Solution

$$
\begin{aligned}
I_1&: \quad (8\ \Omega + 6\ \Omega + 2\ \Omega)I_1 - (2\ \Omega)I_2 = 4\ \text{V} \\
I_2&: \quad (7\ \Omega + 2\ \Omega)I_2 - (2\ \Omega)I_1 = -9\ \text{V}
\end{aligned}
$$

and

$$
\begin{aligned}
16I_1 - 2I_2 &= 4 \\
9I_2 - 2I_1 &= -9
\end{aligned}
$$

which, for determinants, are

$$
\begin{aligned}
16I_1 - 2I_2 &= 4 \\
-2I_1 + 9I_2 &= -9
\end{aligned}
$$

and

$$
I_2 = I_{7\Omega} = \frac{\begin{vmatrix} 16 & 4 \\ -2 & -9 \end{vmatrix}}{\begin{vmatrix} 16 & -2 \\ -2 & 9 \end{vmatrix}} = \frac{-144 + 8}{144 - 4} = \frac{-136}{140}
$$

$$
= \mathbf{-0.971\ A}
$$

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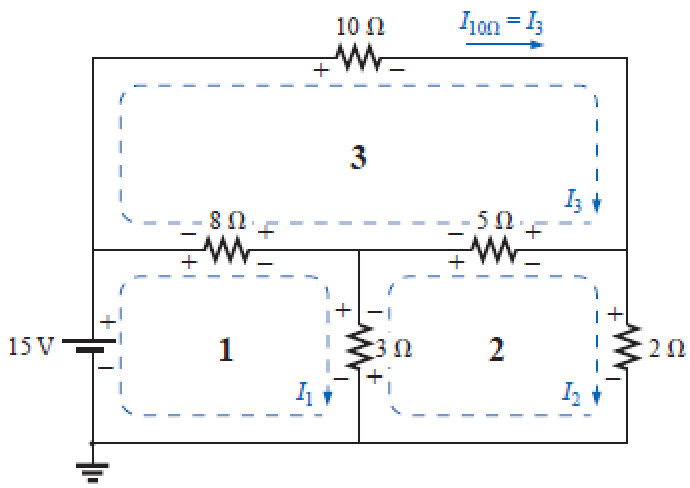
EXAMPLE 2.11 Find the current through the 10-Ω resistor of the network of Fig. below .



Solution:

$$
\begin{aligned}
I_1: &\quad (8\ \Omega + 3\ \Omega)I_1 - (8\ \Omega)I_3 - (3\ \Omega)I_2 = 15\ \text{V} \\
I_2: &\quad (3\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (3\ \Omega)I_1 - (5\ \Omega)I_3 = 0 \\
I_3: &\quad (8\ \Omega + 10\ \Omega + 5\ \Omega)I_3 - (8\ \Omega)I_1 - (5\ \Omega)I_2 = 0
\end{aligned}
$$

$$
\begin{aligned}
11I_1 - 8I_3 - 3I_2 &= 15 \\
10I_2 - 3I_1 - 5I_3 &= 0 \\
23I_3 - 8I_1 - 5I_2 &= 0
\end{aligned}
$$

or

$$
\begin{aligned}
11I_1 - 3I_2 - 8I_3 &= 15 \\
-3I_1 + 10I_2 - 5I_3 &= 0 \\
-8I_1 - 5I_2 + 23I_3 &= 0
\end{aligned}
$$

and

$$
I_3 = I_{10\Omega} = \frac{\begin{vmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{vmatrix}} = \mathbf{1.220\ A}
$$

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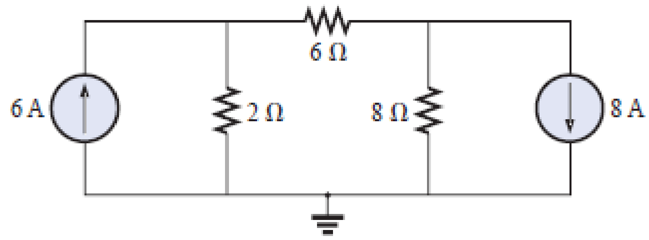
EXAMPLE 2.12 Using mesh analysis, determine the currents for the network of Fig. below .



Solution:

$$I_1 = 6\text{ A}$$
$$I_3 = 8\text{ A}$$

results in the following solutions:

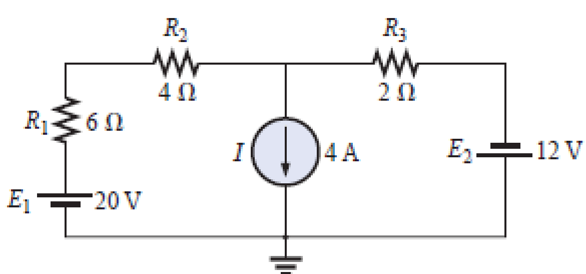
$$2I_1 - 16I_2 + 8I_3 = 0$$
$$2(6\text{ A}) - 16I_2 + 8(8\text{ A}) = 0$$

and $$I_2 = \frac{76\text{ A}}{16} = \mathbf{4.75\text{ A}}$$

Then $$I_{2\Omega}\downarrow = I_1 - I_2 = 6\text{ A} - 4.75\text{ A} = \mathbf{1.25\text{ A}}$$

and $$I_{8\Omega}\uparrow = I_3 - I_2 = 8\text{ A} - 4.75\text{ A} = \mathbf{3.25\text{ A}}$$

EXAMPLE 2.13 Using mesh analysis, determine the currents of the network of Fig. below.



Solution:

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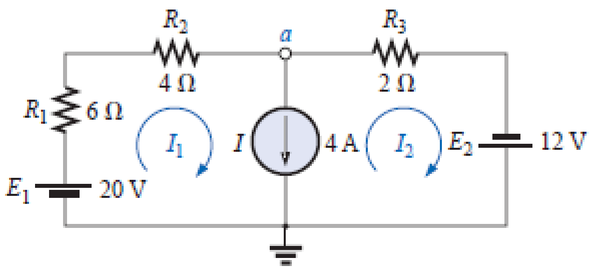

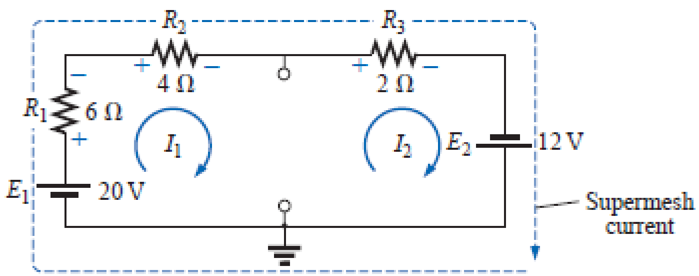

Applying Kirchhoff's law:

$$20\text{ V} - I_1(6\ \Omega) - I_1(4\ \Omega) - I_2(2\ \Omega) + 12\text{ V} = 0$$

or
$$10I_1 + 2I_2 = 32$$

Node a is then used to relate the mesh currents and the current source using Kirchhoff's current law:

$$I_1 = I + I_2$$

The result is two equations and two unknowns:

$$\begin{aligned} 10I_1 + 2I_2 &= 32 \\ I_1 - \ \ I_2 &= 4 \end{aligned}$$

Applying determinants:

$$I_1 = \frac{\begin{vmatrix} 32 & 2 \\ 4 & -1 \end{vmatrix}}{\begin{vmatrix} 10 & 2 \\ 1 & -1 \end{vmatrix}} = \frac{(32)(-1) - (2)(4)}{(10)(-1) - (2)(1)} = \frac{40}{12} = \mathbf{3.33\ A}$$

and
$$I_2 = I_1 - I = 3.33\text{ A} - 4\text{ A} = \mathbf{-0.67\ A}$$